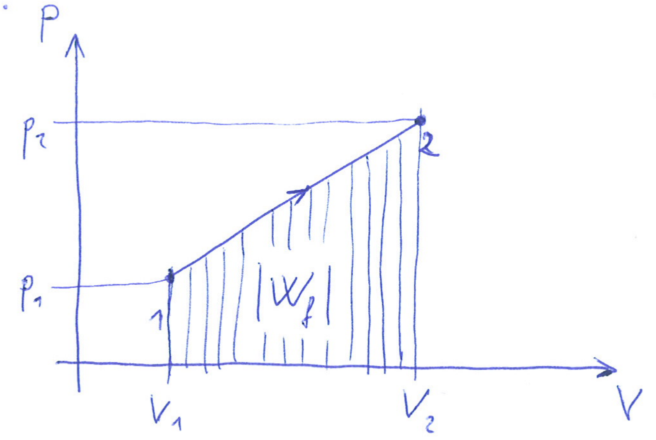
BGTD – 1. Matematikai bevezető

1.1.



$p_1 = 1[bar]$ $\qquad V_1 = 0{,}2[m^3]$

$p_2 = 3[bar]$ $\qquad V_2 = 0{,}6[m^3]$

$$W_f = -\int_{V_1}^{V_2} p(V)\,dV$$

$$p(V) = aV + b$$

$$p(V_1) = \boxed{p_1 = aV_1 + b} \quad (1)$$

$$p(V_2) = \boxed{p_2 = aV_2 + b} \quad (2)$$

$$(2)-(1): \quad p_2 - p_1 = a(V_2 - V_1) \Rightarrow a = \frac{p_2 - p_1}{V_2 - V_1} = 5\cdot 10^5 \left[\frac{Pa}{m^3}\right]$$

$$(2)\cdot V_1 - (1)\cdot V_2: \quad p_2 V_1 - p_1 V_2 = b(V_1 - V_2) \Rightarrow b = \frac{p_2 V_1 - p_1 V_2}{V_1 - V_2} = 0[Pa]$$

$$p(V) = aV = 5\cdot 10^5 \left[\frac{Pa}{m^3}\right]\cdot V$$

$$W_f = -\int_{V_1}^{V_2} p(V)\,dV = -\int_{V_1}^{V_2} aV\,dV = -\frac{a}{2}\left[V^2\right]_{V_1}^{V_2} = -\frac{a}{2}\left[V_2^2 - V_1^2\right] =$$

$$= -80[kJ]$$

W_f, mint trapéz területe:

$$|W_f| = p_1(V_2 - V_1) + \frac{(p_2 - p_1)(V_2 - V_1)}{2} = \frac{p_1 + p_2}{2}(V_2 - V_1) = 80[kJ]$$

előjel: tágulás $\rightsquigarrow$ a gáz végez munkát $\rightsquigarrow$ $\ominus$

összenyomás/kompresszió $\rightsquigarrow$ a gázon végzünk munkát $\rightsquigarrow$ $\oplus$

1.2. $pV^n = \text{áll}$ (vagy $pv^n = \text{áll}$)

↳ politropikus állapotváltozás, n - politropikus kitevő,

$n \in (-\infty, +\infty)$ ⇝ papíron!

(1) $pv^n = \text{áll}$ ⇝ politropikus állapotváltozás

(2) $\frac{pv}{T} = \text{áll}$ ⇝ ideális gáz állapotegyenletéből

$\frac{(1)}{(2)}$: $Tv^{n-1} = \text{áll}$

$\frac{(1)}{(2)^n}$: $T^n p^{1-n} = \text{áll}$ ⇝ $Tp^{\frac{1-n}{n}} = \text{áll}$

speciális esetek:

$n = 0$ ⇝ izobár

$n = 1$ ⇝ izoterm

$n = \kappa$ ⇝ adiabatikus (nincs hőközlés) + reverzibilis ⇒ izentropikus

$n \to \infty$ ⇝ izochor

$$W_f = -\int_{v_1}^{v_2} p(v)\,dv = -p_1 v_1^n \int_{v_1}^{v_2} v^{-n}\,dv \overset{n=1}{=} -p_1 v_1 \int_{v_1}^{v_2} \frac{dv}{v} = -p_1 v_1 \ln\frac{v_2}{v_1} = p_1 v_1 \ln\frac{p_2}{p_1}$$

$pv^n = p_1 v_1^n \Rightarrow p(v) = p_1 v_1^n v^{-n}$

$\frac{v_2}{v_1} = \frac{p_1}{p_2}$

$$\overset{n\neq 1}{=} -p_1 v_1^n \left[\frac{v^{1-n}}{1-n}\right]_{v_1}^{v_2} = -\frac{p_1 v_1^n}{1-n}\left(v_2^{1-n} - v_1^{1-n}\right) =$$

$$= -\frac{p_1}{1-n}\left[\left(\frac{v_2}{v_1}\right)^{1-n} v_1 - \left(\frac{v_1}{v_1}\right)^{1-n} v_1\right] = \frac{p_1 v_1}{n-1}\left[\left(\frac{v_2}{v_1}\right)^{1-n} - 1\right] = \frac{p_1 v_1}{n-1}\left[\left(\frac{v_1}{v_2}\right)^{n-1} - 1\right]$$

$$W_t = \int_{p_1}^{p_2} v(p)\,dp = p_1^{\frac{1}{n}} v_1 \int_{p_1}^{p_2} p^{-\frac{1}{n}}\,dp \overset{n=1}{=} p_1 v_1 \int_{p_1}^{p_2} \frac{dp}{p} = p_1 v_1 \ln\frac{p_2}{p_1}$$

$$p v^n = p_1 v_1^n \Rightarrow v(p) = p_1^{\frac{1}{n}} v_1\, p^{-\frac{1}{n}}$$

$$\overset{n\neq 1}{=} p_1^{\frac{1}{n}} v_1 \left[\frac{p^{1-\frac{1}{n}}}{1-\frac{1}{n}}\right]_{p_1}^{p_2} = \frac{p_1^{\frac{1}{n}} v_1}{\frac{n-1}{n}}\left[p_2^{\frac{n-1}{n}} - p_1^{\frac{n-1}{n}}\right] =$$

$$= \frac{n}{n-1} v_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} p_1 - \left(\frac{p_1}{p_1}\right)^{\frac{n-1}{n}} p_1\right] = \frac{n}{n-1} p_1 v_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right] =$$

$$= \frac{n}{n-1} p_1 v_1 \left[\left(\frac{p_1}{p_2}\right)^{\frac{1-n}{n}} - 1\right] = \frac{n}{n-1} p_1 v_1 \left\{\left[\left(\frac{v_1}{v_2}\right)^{-n}\right]^{\frac{1-n}{n}} - 1\right\} = \frac{n}{n-1} p_1 v_1 \left[\left(\frac{v_1}{v_2}\right)^{n-1} - 1\right]$$

$$p_1 v_1^n = p_2 v_2^n \Rightarrow \frac{p_1}{p_2} = \left(\frac{v_2}{v_1}\right)^n$$

- $V_1 = 0{,}5\,[m^3]$ $\quad p_1 = 1\,[bar]$ $\quad V_2 = 0{,}2\,[m^3]$

$n = 0$: ↓ izobár

$$W_f = -\int_{V_1}^{V_2} p\,dV = -p_1 (V_2 - V_1) = 30\,[kJ]$$

$$W_t = \int_{p_1}^{p_2} V\,dp \underset{p_1 = p_2}{=} 0$$

$n = 1$: ↓ izoterm

$$W_f = -\int_{V_1}^{V_2} p\,dV \underset{p_1 V_1 = pV}{=} -p_1 V_1 \int_{V_1}^{V_2} \frac{dV}{V} = -p_1 V_1 \ln\frac{V_2}{V_1} = 45{,}8145\,[kJ]$$

$$W_t = \int_{p_1}^{p_2} V\,dp = p_1 V_1 \int_{p_1}^{p_2} \frac{dp}{p} = p_1 V_1 \ln\frac{p_2}{p_1} = p_1 V_1 \ln\frac{V_1}{V_2} = W_f$$

$$p_1 V_1 = p_2 V_2 \Rightarrow \frac{p_2}{p_1} = \frac{V_1}{V_2}$$

$n = \kappa$: ↓ izentropikus (ld. később)

$$W_f = \frac{p_1 V_1}{\kappa - 1}\left[\left(\frac{V_1}{V_2}\right)^{\kappa - 1} - 1\right] = 55{,}3375\,[kJ]$$

$$W_t = \frac{\kappa}{\kappa - 1} p_1 V_1 \left[\left(\frac{V_1}{V_2}\right)^{\kappa - 1} - 1\right] = 77{,}4725\,[kJ]$$

általános képletbe behelyettesítve

↳ másképp: $p_2 = p_1 \left(\frac{V_1}{V_2}\right)^{\kappa} = 3{,}6067\,[bar]$, ebből a p-s képletbe helyettesítve

- $V_1 = 0{,}5\,[m^3]$ $\quad p_1 = 1[bar]$ $\quad p_2 = 2[bar]$

$n \to \infty$: $\quad W_f = -\int_{V_1}^{V_2} p\,dV \underset{V_1=V_2}{=} 0$

$\downarrow$

izochor

$$W_t = \int_{p_1}^{p_2} V\,dp = V(p_2 - p_1) = 50\,[kJ]$$

közölt hő (ideális gáz esetén)

$$du = dq + dw_f \Rightarrow dq = du - dw_f \Big) \int_1^2$$

// ideális gázra: $du = c_v\,dT$

további: $c_v = konst$

(tökéletes gáz) //

$$q = u_2 - u_1 - w_f$$

$$q = c_v(T_2 - T_1) - w_f \overset{n=1 \to \text{izoterm}}{=} -w_f(n=1) = -p_1 v_1 \ln\frac{v_2}{v_1} \underset{\substack{p_1 v_1 = RT_1 \\ \text{(ideális gázra)}}}{=}$$

$$= -RT_1 \ln\frac{v_2}{v_1} = -RT_1 \ln\frac{p_2}{p_1}$$

$$\overset{n\neq 1}{=} c_v(T_2 - T_1) - \frac{p_1 v_1}{n-1}\left[\left(\frac{v_1}{v_2}\right)^{n-1} - 1\right] \underset{\substack{p_1 v_1 = RT_1 \\ \left(\frac{v_1}{v_2}\right)^{n-1} = \frac{T_2}{T_1}}}{=} c_v(T_2 - T_1) - \frac{RT_1}{n-1}\left[\frac{T_2}{T_1} - 1\right] =$$

ideális gázra

$$= \left(c_v - \frac{R}{n-1}\right)(T_2 - T_1) = c_n(T_2 - T_1)$$

$\hookrightarrow$ politropikus fajhő

$n = 0$: $\quad c_0 = c_v - \frac{R}{-1} = c_v + R = c_p$ $\quad$ (ideális gázra)

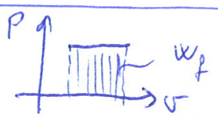
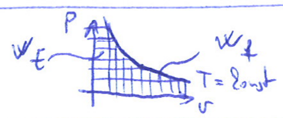
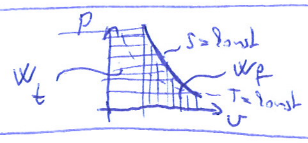
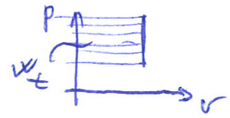
$n = 1$ $\quad c_1 = c_v - \frac{R}{0}$ $\rightsquigarrow$ nincs értelme, külön esetben kezeltük

$n = \kappa$ $\quad c_\kappa = c_v - \frac{R}{\kappa - 1} = \frac{c_v(\kappa-1) - R}{\kappa - 1} = \frac{c_v\kappa - c_v - R}{\kappa - 1} \underset{\frac{c_p}{c_v} = \kappa,\ R = c_p - c_v}{=} 0$

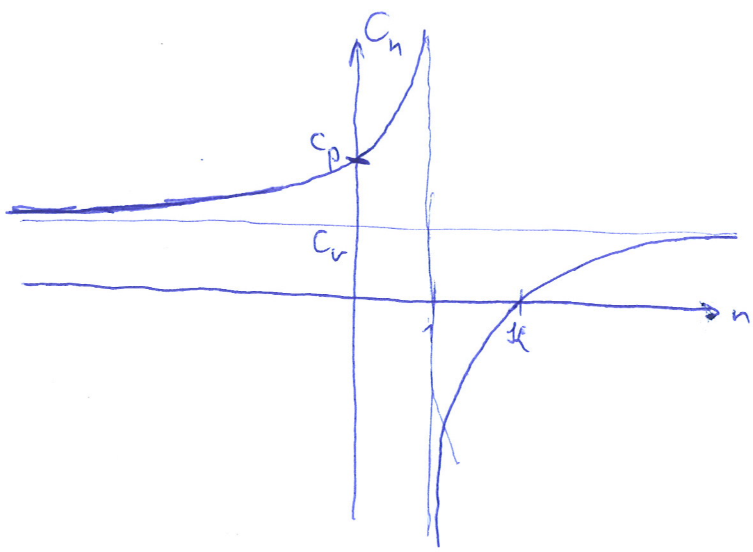
$\hookrightarrow$ $n = \kappa$ esetén a közölt hő $0 \Rightarrow$ adiabatikus, ha reverzibilis is izentropikus állapotváltozás

$n \to \infty$ $\quad c_\infty = c_v - \lim_{n\to\infty}\frac{R}{n-1} = c_v$

n	állapotváltozás	w_f	w_t
0	izobár	$-p_1(v_2-v_1)$	0
1	izoterm	$p_1v_1\ln\frac{v_1}{v_2}=p_1v_1\ln\frac{p_2}{p_1}$	$w_t=w_f$
κ	adiabatikus + reverzibilis = izentropikus	$\frac{p_1v_1}{\kappa-1}\left[\left(\frac{v_1}{v_2}\right)^{\kappa-1}-1\right]$	$\frac{\kappa}{\kappa-1}p_1v_1\left[\left(\frac{v_1}{v_2}\right)^{\kappa-1}-1\right]$
∞	izochor	0	$v_1(p_2-p_1)$

n	q	c_n	(p-v) diagramm
0	$c_p(T_2-T_1)$	c_p	
1	$-p_1v_1\ln\frac{v_2}{v_1}=-RT_1\ln\frac{p_2}{p_1}$	„∞"	
κ	0	0	
∞	$c_v(T_2-T_1)$	c_v	

c_n fajhő az n politropikus kitevő függvényében



1.3.

$p = 200\,[kPa] = \text{konst}$ $\qquad M = 28{,}97\left[\frac{kg}{kmol}\right]$

$T_1 = 300\,[K] \qquad T_2 = 600\,[K]$

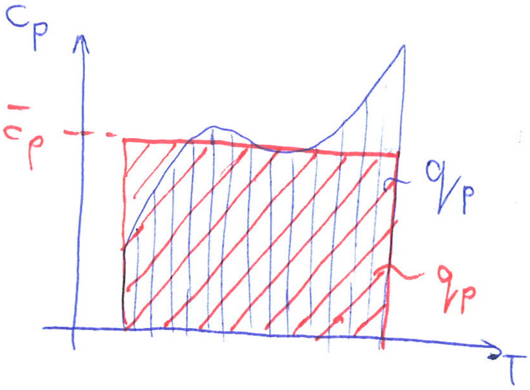
$$c_{p,m}(T) = a + bT + cT^2 + dT^4 \quad \left[\frac{kJ}{kmol\,K}\right]$$

$$a = 28{,}11\left[\frac{kJ}{kmol\,K}\right] \qquad b = 0{,}1967\cdot 10^{-2}\left[\frac{kJ}{kmol\,K^2}\right]$$

$$c = 0{,}4802\cdot 10^{-5}\left[\frac{kJ}{kmol\,K^3}\right] \qquad d = -1{,}966\cdot 10^{-9}\left[\frac{kJ}{kmol\,K^4}\right]$$

$$\left[\frac{kJ}{kg}\right] \underset{\substack{\uparrow \\ \text{izobár}}}{q_p} = \frac{q_{p,m}}{M}\ \frac{\left[\frac{kJ}{kmol}\right]}{\left[\frac{kg}{kmol}\right]} = \frac{1}{M}\int_{T_1}^{T_2} c_{p,m}(T)\,dT =$$

$$= \frac{1}{M}\left[aT + \frac{b}{2}T^2 + \frac{c}{3}T^3 + \frac{d}{4}T^4\right]_{T_1}^{T_2} = 308{,}6418\left[\frac{kJ}{kg}\right]$$



átlagot úgy határozzuk meg, hogy a két hőmérséklethatár között a közölt hőt egyeztetjük meg

$$q_p = \bar{c}_p(T_2 - T_1) = \frac{1}{M}\int_{T_1}^{T_2} c_{p,m}(T)\,dT \Rightarrow \bar{c}_p = \frac{\frac{1}{M}\int_{T_1}^{T_2} c_{p,m}(T)\,dT}{T_2 - T_1} =$$

$$= 1028{,}806\left[\frac{J}{kg\,K}\right]$$

$$\bar{c}_{p,m} = \frac{\int_{T_1}^{T_2} c_{p,m}(T)\,dT}{T_2 - T_1} = \bar{c}_p\cdot M = 29{,}8045\left[\frac{kJ}{kmol\,K}\right]$$