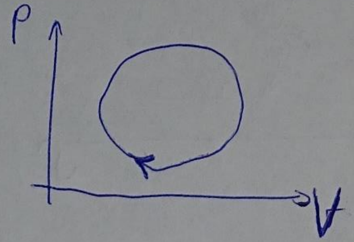
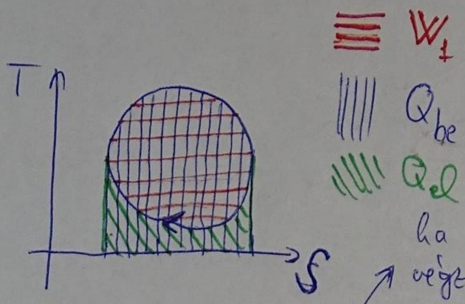
# BGTD – 7. Gázkörfolyamatok

## Körfolyamat:




ha a rendszer végzi a munkát ⊖, így $-\ominus = \oplus \Rightarrow$ ezt nyerjük

$$\underbrace{U_2 - U_1}_{=0} = Q_{12} + W_{t,12} \Rightarrow Q_{12} = -W_{t,12}$$

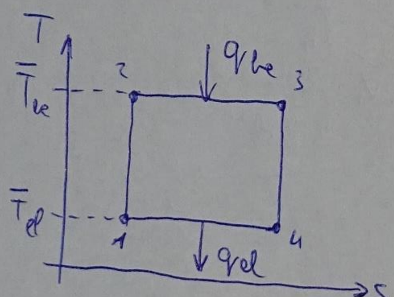
$=0$ // $U_2 = U_1$ / a kezdő és végpont megegyezik, mivel $U$ állapotjelző, így $U_1 = U_2$ //

$$|W_{t,12}| = |Q_{12}| = |Q_{be}| - |Q_{el}|$$

termikus hatásfok: $$\eta_{th} = \frac{|Q_{be}| - |Q_{el}|}{|Q_{be}|} = \frac{\overset{\text{hasznos}}{|W_{t,12}|}}{\underset{\text{befektetett}}{|Q_{be}|}} = 1 - \frac{|Q_{el}|}{|Q_{be}|}$$

## Carnot-körfolyamat:

Legjobb termikus hatásfokú körfolyamat, elméleti maximumot ad




1 → 2 adiabatikus + reverzibilis „kompresszió"

2 → 3 izoterm hőbevezetés

3 → 4 adiabatikus + reverzibilis „expanzió"

4 → 1 izoterm hőelvonás

$$\eta_{th,C} = \frac{|W|}{|Q_{be}|} = 1 - \frac{|Q_{el}|}{|Q_{be}|} = 1 - \frac{\bar{T}_{el}(s_4 - s_1)}{\bar{T}_{be}(s_3 - s_2)} = 1 - \frac{\bar{T}_{el}}{\bar{T}_{be}}$$

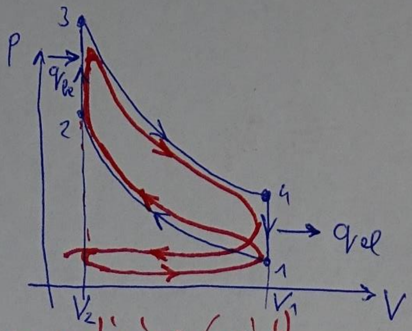
$s_3 = s_4$

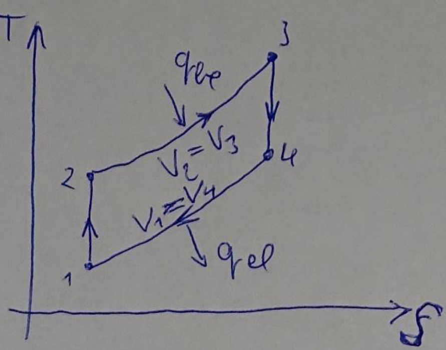
$s_2 = s_1$

hőbevezetés és hőelvonás termodinamikai középhőmérséklete

$$\eta_{th,C} > \eta_{th}$$

## Otto-körfolyamat: szikragyújtású motorok helyettesítő körfolyamata




— valóságos (mért) indikátordiagramm

1→2 adiabatikus + reverzibilis kompresszió

2→3 izochor hőbevezetés

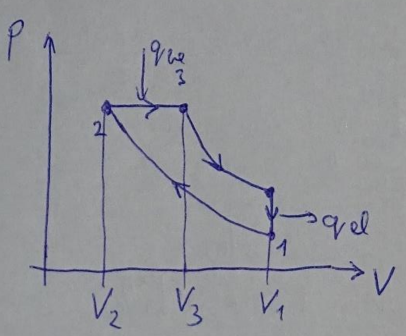
3→4 adiabatikus + reverzibilis expanzió

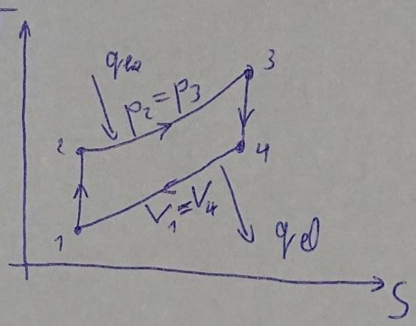
4→1 izochor hőelvonás

$$\varepsilon = r_V = \frac{V_1}{V_2} \text{ - kompresszióviszony} \qquad \varepsilon_{Otto} = 8 \div 11$$

$$\eta_{th} = 1 - \frac{1}{\varepsilon^{\kappa-1}}$$

## Diesel-körfolyamat: kompressziógyújtású motorok helyettesítő körfolyamata




1→2 adiabatikus + reverzibilis kompresszió

2→3 ~~adiabatikus~~ izobár hőbevezetés

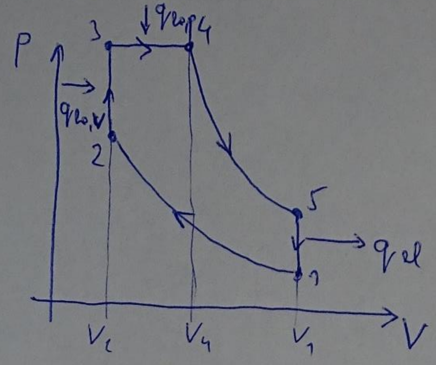
3→4 adiabatikus + reverzibilis expanzió

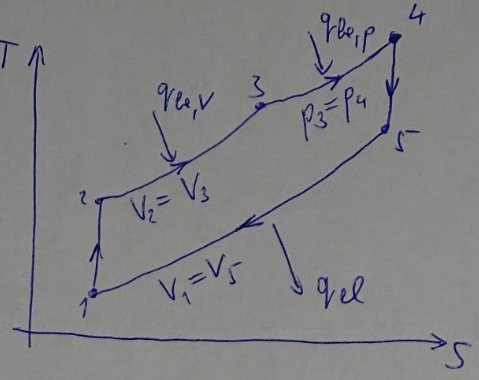
4→1 izochor hőelvonás

$$\varphi = r_\varepsilon = \frac{V_3}{V_2} \text{ - előzetes expanzióviszony} \qquad \varepsilon_{Diesel} = 12 \div 23$$

$$\eta_{th} = 1 - \frac{1}{\varepsilon^{\kappa-1}} \cdot \frac{\varphi^{\kappa} - 1}{\kappa(\varphi - 1)}$$

## Seiliger – Sabathé – körfolyamat: módosított Diesel – körfolyamat




1 → 2 adiabatikus + reverzibilis kompresszió

2 → 3 izochor hőbevezetés

3 → 4 izobár hőbevezetés

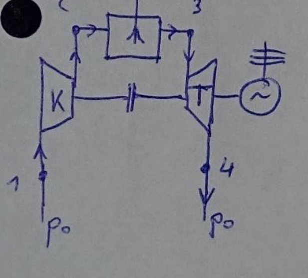
4 → 5 adiabatikus + reverzibilis expanzió

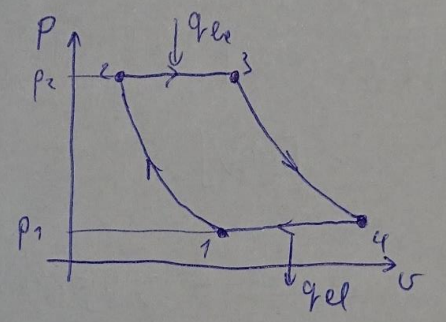
5 → 1 izochor hőelvonás

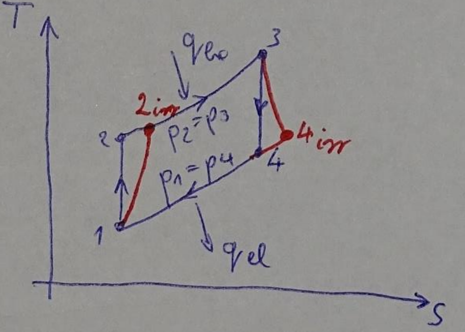
Belsőégésű motorokra jellemző effektív középnyomás (Mean Effective Pressure = MEP):

$$MEP = \frac{|W|}{V_{max} - V_{min}}$$

## Joule – Brayton – körfolyamat: gázturbina helyettesítő körfolyamata




1→2: adiabatikus (+ reverzibilis) kompresszió

2→3: izobár hőbevezetés

3→4: adiabatikus (+ reverzibilis) expanzió

4→1: izobár hőelvonás

$\pi = r_p = \frac{p_2}{p_1}$ – nyomásviszony $\qquad \eta_{th} = 1 - \frac{1}{\pi^{\frac{\kappa-1}{\kappa}}}$

optimális nyomásviszony (hasznos munka szempontjából): $\pi_{opt} = \left(\frac{T_3}{T_1}\right)^{\frac{\kappa}{2(\kappa-1)}}$

7.3.1. $\dot{Q}_{el} = 420$ [MW]   $P_H = \dot{W}_H = 220$ [MW]

$\Delta \dot{S}_{el} = 1250 \left[\frac{kW}{K}\right]$

$$\dot{Q}_{be} = P_H + \dot{Q}_{el} = 640 \text{ [MW]}$$

$$\eta_{th} = \frac{|\dot{W}_H|}{|\dot{Q}_{be}|} = 0{,}34375 \text{ [1]}$$

egyenértékű Carnot körfolyamat: $\eta_{th} = \eta_{th,C}$

$$\bar{T}_{el} = \frac{\dot{Q}_{el}}{\Delta \dot{S}_{el}} = 336 \text{ [K]}$$

$$\eta_{th} = \eta_{th,C} = 1 - \frac{\bar{T}_{el}}{\bar{T}_{be}} \Rightarrow \bar{T}_{be} = \frac{\bar{T}_{el}}{1 - \eta_{th,C}} = 512 \text{ [K]}$$

7.3.2. $\bar{T}_{be} = 450$ [K]

$\Delta \dot{S}_{be} = 2 \left[\frac{MW}{K}\right]$   $\Delta \dot{S}_{el} = 2{,}25 \left[\frac{MW}{K}\right]$

$\dot{Q}_{el} = 675$ [MW]

$$\dot{Q}_{be} = \bar{T}_{be} \, \Delta \dot{S}_{be} = 900 \text{ [MW]}$$

$$\dot{W}_H = \dot{Q}_{be} - \dot{Q}_{el} = 225 \text{ [MW]}$$

$$\eta_{th} = \frac{|\dot{Q}_{be}| - |\dot{Q}_{el}|}{|\dot{Q}_{be}|} = 1 - \frac{|\dot{Q}_{el}|}{|\dot{Q}_{be}|} = 0{,}25 \text{ [1]}$$

~~ez most éppen Carnot körfolyamat (konstans hőbevezetési és hőelvezetési hőmérséklet)~~

$$\bar{T}_{el} = \frac{\dot{Q}_{el}}{\Delta \dot{S}_{el}} = 300 \text{ [K]}$$

7.3.3. $\bar{T}_{be} = 480\ [K]$ $\dot{Q}_{be} = 600\ [MW]$

$\dot{Q}_{el} = 400\ [MW]$

$$\eta_{th} = 1 - \frac{|\dot{Q}_{el}|}{|\dot{Q}_{be}|} = 0{,}33\ [1]$$

$$\eta_{th} \underset{\text{most}}{=} \eta_{th,C} = 1 - \frac{\bar{T}_{el}}{\bar{T}_{be}} \Rightarrow \bar{T}_{el} = \bar{T}_{be}(1-\eta_{th}) = 320\ [K]$$

$$\bar{T}_{el}^{*} = \bar{T}_{el} + 5\ [K] = 325\ [K]$$

$$\eta_{th}^{*} = 1 - \frac{\bar{T}_{el}^{*}}{\bar{T}_{be}} = 0{,}3229\ [1]$$ $\rightsquigarrow$ 1,04%-kal csökken a hatásfok

7.3.4. $\Delta\dot{S}_{el} = 2\left[\frac{MW}{K}\right]$ $\bar{T}_{be} = 450\,K$ $\eta_{th} = \frac{1}{3}\ [1]$

egyensúlyi Carnot-körfolyamat: $\eta_{th} = \eta_{th,C}$

$$\eta_{th,C} = 1 - \frac{\bar{T}_{el}}{\bar{T}_{be}} \Rightarrow \bar{T}_{el} = \bar{T}_{be}(1-\eta_{th,C}) = 300\,K$$

$\Delta\dot{S}_{el} = \Delta\dot{S}_{be}$ (Carnot-esetén)

$$\dot{Q}_{be} = \bar{T}_{be}\,\Delta\dot{S}_{be} = 900\ [MW]$$

$$\dot{Q}_{el} = \bar{T}_{el}\,\Delta\dot{S}_{be} = 600\ [MW]$$

$$|\dot{W}_{H}| = |\dot{Q}_{be}| - |\dot{Q}_{el}| = 300\ [MW]$$

7.3.5. $\bar{T}_{be} = 460\ [K]$ $\eta_{th} = 35\ [\%]$ $\dot{Q}_{el} = 600\ [MW]$

$\bar{T}_{ir} = 400\ [K]$ $\Delta\dot{S}_{ir} = 20\left[\frac{kW}{K}\right]$

$$\eta_{th} = 1 - \frac{|\dot{Q}_{el}|}{|\dot{Q}_{be}|} \Rightarrow \dot{Q}_{be} = \frac{|\dot{Q}_{el}|}{1-\eta_{th}} = 923{,}077\ [K]$$

$$\parallel$$

$$\eta_{th,C} = 1 - \frac{\bar{T}_{el}}{\bar{T}_{be}} \Rightarrow \bar{T}_{el} = \bar{T}_{be}(1-\eta_{th,C}) = 299\ [K]$$

$$\dot{Q}^*_{el} = \dot{Q}_{el} + \bar{T}_{el} \cdot \Delta \dot{S}_{irr} = 605{,}98 \text{ [MW]}$$

$$|\dot{W}_H| = |\dot{Q}_{be}| - |\dot{Q}_{el}| = 317{,}097 \text{ [MW]}$$

7.4.1. $m = 200 \text{ [g]}$ $\quad M_{CO_2} = 44 \left[\frac{kg}{kmol}\right]$ $\quad \varkappa = 1{,}3 \text{ [1]}$

$p_1 = 15 \text{[bar]}$ $\quad T_1 = 20 [^\circ C]$

$$R = \frac{\mathring{R}}{M_{CO_2}} = 188{,}964 \left[\frac{J}{kg\,K}\right] \qquad c_p = \frac{\varkappa R}{\varkappa - 1} = 818{,}844 \left[\frac{J}{kg\,K}\right]$$

$$c_v = \frac{R}{\varkappa-1} = 629{,}88 \left[\frac{J}{kg\,K}\right]$$

$1 \to 2$: adiabatikus, reverzibilis kompresszió $p_2 = 40 \text{ [bar]}$-ra:

$$T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{\frac{\varkappa-1}{\varkappa}} = 367{,}424 \text{ [K]}$$

$Q_{12} = 0$ (adiabatikus)

$$U_2 - U_1 = W_{12} + \underbrace{Q_{12}}_{=0} \Rightarrow W_{12} = U_2 - U_1 = c_v m (T_2 - T_1) =$$
$$= 9375{,}64 \text{ [J]}$$

$2 \to 3$: politropikus expanzió: $n = 1{,}2 \text{ [1]}$:

$$Q_{23} = c_n m (T_3 - T_2) = c_v \frac{n-\varkappa}{n-1} m (T_3 - T_2) = 3490{,}23 \text{ [J]}$$

$$T_3 = T_2 \left(\frac{p_3}{p_2}\right)^{\frac{n-1}{n}} = 312{,}013 \text{ [K]}$$

$p_3 = p_1$

$$U_3 - U_2 = W_{23} + Q_{23} \Rightarrow W_{23} = (U_3 - U_2) - Q_{23} =$$
$$= c_v m (T_3 - T_2) - c_v \frac{n-\varkappa}{n-1} m (T_3 - T_2) =$$
$$= c_v m (T_3 - T_2) \left[1 - \frac{n-\varkappa}{n-1}\right] = -10\,470{,}7 \text{ [J]}$$

3 → 1: izobár hőelvonás:

$$Q_{31} = c_p m (T_1 - T_3) = -3113{,}74 \; [J]$$

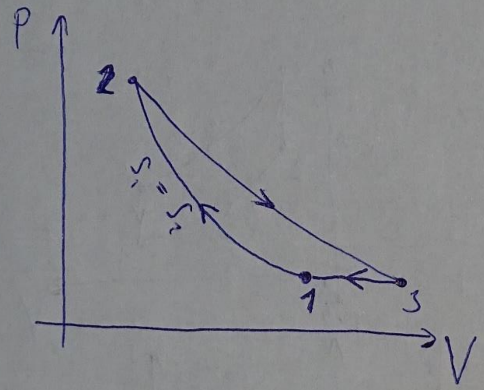
$$U_1 - U_3 = Q_{31} + W_{31} \Rightarrow W_{31} = (U_1 - U_3) - Q_{31} =$$

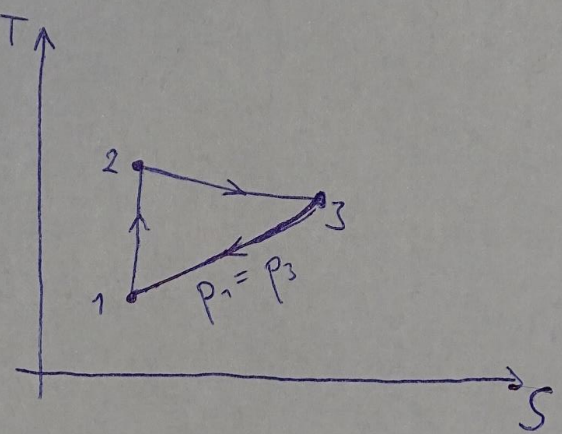
$$= c_v m (T_1 - T_3) - c_p m (T_1 - T_3) =$$

$$= \underbrace{(c_v - c_p)}_{-R} m (T_1 - T_3) =$$

$$= -m R (T_1 - T_3) = 718{,}555 \; [J]$$

$$\eta_{th} = \frac{|W|}{|Q_{be}|} = \frac{|W_{12} + W_{23} + W_{31}|}{|Q_{23}|} = 0{,}185032 \; [1]$$




7.4.2. $\varepsilon = \dfrac{V_1}{V_2} = 11{,}5 \; [1]$

$p_1 = 1{,}02$ [bar] $\quad T_1 = 32$ [°C] $\quad \dot{m} = 800 \left[\frac{kg}{h}\right]$

$p_3 = 42$ [bar] $\quad R = 287 \left[\frac{J}{kgK}\right] \quad \kappa = 1{,}4 \; [1]$

$$c_v = \frac{R}{\kappa - 1} = 717{,}5 \left[\frac{J}{kgK}\right]$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\varkappa-1} = \varepsilon^{\varkappa-1} \Rightarrow T_2 = T_1\,\varepsilon^{\varkappa-1} = 810,175 \ [K]$$

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\varkappa-1}{\varkappa}} \Rightarrow p_2 = p_1\left(\frac{T_2}{T_1}\right)^{\frac{\varkappa}{\varkappa-1}} = p_1\left[\left(\underbrace{\frac{V_1}{V_2}}_{\varepsilon}\right)^{\varkappa-1}\right]^{\frac{\varkappa}{\varkappa-1}} = p_1\,\varepsilon^{\varkappa} =$$

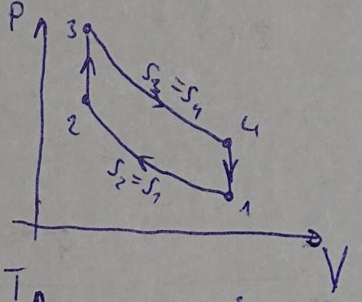
$$= 31,1585 \ [bar]$$

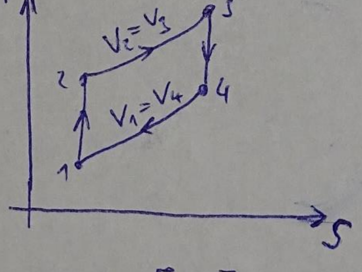
izochor hőbevezetés után a hőmérséklet:

$$R = \frac{p_2 V_2}{m T_2} = \frac{p_3 V_3}{m T_3} \Rightarrow T_3 = T_2 \frac{p_3}{p_2} \underbrace{\frac{V_3}{V_2}}_{=1 \ /\text{izochor}} = 1092,07 \ [K]$$

$$\frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{\varkappa-1} = \left(\frac{V_2}{V_1}\right)^{\varkappa-1} = \frac{1}{\varepsilon^{\varkappa-1}} \Rightarrow T_4 = T_3\frac{1}{\varepsilon^{\varkappa-1}} = 411,123 \ [K]$$

$V_3 = V_2$
$V_4 = V_1$




$$q_{be} = c_v\,(T_3 - T_2) = 202,260 \left[\frac{kJ}{kg}\right]$$

$$q_{el} = c_v\,(T_1 - T_4) = -76,1433 \left[\frac{kJ}{kg}\right]$$

$$|w| = |q_{be}| - |q_{el}| = 126,117 \left[\frac{kJ}{kg}\right]$$

$$\dot{P} = |\dot{W}_H| = \dot{m}\,|w| = 28,026 \ [kW] \cong 37,5548 \ [hp]$$

$$\eta = \frac{|w|}{|q_{be}|} = 0,6235 \ [1]$$

7.4.3. $\varepsilon = \frac{V_1}{V_2} = 13{,}6 \; [1]$

$p_3 = 50 \; [bar]$

$V_4 = 2V_3 \Rightarrow \varphi = \frac{V_4}{V_3} = 2 \; [1]$

$T_1 = 15 \; [°C] \qquad p_1 = 1 \; [bar]$

$R = 287 \left[\frac{J}{kgK}\right]$

$\varkappa = 1{,}4 \; [1]$

$\dot{m} = 800 \left[\frac{kg}{h}\right]$

$$c_v = \frac{R}{\varkappa - 1} = 717{,}5 \left[\frac{J}{kgK}\right] \qquad c_p = c_v \varkappa = 1004{,}5 \left[\frac{J}{kgK}\right]$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\varkappa-1} = \varepsilon^{\varkappa-1} \Rightarrow T_2 = T_1 \varepsilon^{\varkappa-1} = 818{,}103 \; [K]$$

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\varkappa-1}{\varkappa}} \Rightarrow p_2 = p_1 \left(\frac{T_2}{T_1}\right)^{\frac{\varkappa}{\varkappa-1}} = p_1 \left[\left(\frac{V_1}{V_2}\right)^{\varkappa-1}\right]^{\frac{\varkappa}{\varkappa-1}} =$$

$$= p_1 \varepsilon^{\varkappa} = 38{,}6326 \; [bar]$$

hőmérséklet azizochor hőbelvezetés után:

$$T_3 = T_2 \frac{p_3}{p_2} = 1058{,}82 \; [K]$$

hőmérséklet az izobár hőbevezetés után:

$$T_4 = T_3 \underbrace{\frac{V_4}{V_3}}_{=\varphi} = 2117{,}65 \; [K]$$

$$\frac{T_5}{T_4} = \left(\frac{V_4}{V_5}\right)^{\varkappa-1} \underset{V_5 = V_1}{=} \left(\underbrace{\frac{V_4}{V_3}}_{\varphi} \underbrace{\frac{V_3}{V_1}}_{\frac{1}{\varepsilon}}\right)^{\varkappa-1} = \left(\frac{\varphi}{\varepsilon}\right)^{\varkappa-1} \Rightarrow T_5 = T_4 \left(\frac{\varphi}{\varepsilon}\right)^{\varkappa-1} = 983{,}673 \; [K]$$

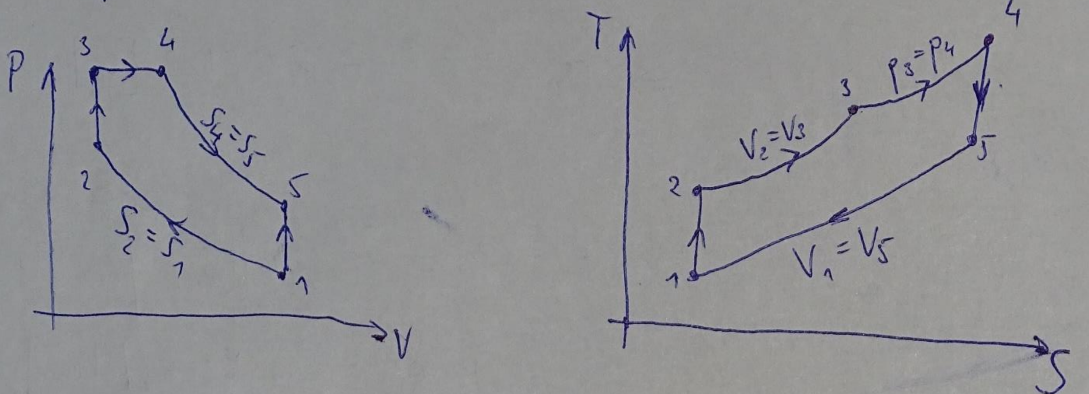
$$q_{be} = q_{be,v} + q_{be,p} = c_v(T_3 - T_2) + c_p(T_4 - T_1) = 1236,31 \left[\frac{kJ}{kg}\right]$$

$$q_{el} = c_v(T_1 - T_5) = -499,145 \left[\frac{kJ}{kg}\right]$$

$$|w| = |q_{be}| - |q_{el}| = 737,165 \left[\frac{kJ}{kg}\right]$$

$$P = |\dot{W}_H| = \dot{m}|w| = 163,814\ [kW] \cong 219,511\ [hp]$$

$$\eta = \frac{|w|}{|q_{be}|} = 0,5962\ [1]$$




7.4.4. $\eta_K = 85\ [\%]$ $\quad \eta_T = 92\ [\%]$

$T_1 = 10\ [°C]$ $\quad p_1 = 1\ [bar]$

$p_2 = 16\ [bar]$ $\quad \dot{m} = 300 \left[\frac{kg}{s}\right]$

$R = 287 \left[\frac{J}{kgK}\right]$ $\quad \varkappa = 1,4\ [1]$

$$c_p = \frac{\varkappa R}{\varkappa - 1} = 1004,5 \left[\frac{J}{kgK}\right]$$

$$\frac{T_{2,rev}}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\varkappa-1}{\varkappa}} \Rightarrow T_{2,rev} = T_1 \underbrace{\left(\frac{p_2}{p_1}\right)}_{\pi}^{\frac{\varkappa-1}{\varkappa}} = 624,915\ [K]$$

összejárásában nincs hasznos teljesítmény, turbina által leadott munka a kompresszor hajtását szolgálja:

$$|\dot{W}_K| = |\dot{W}_T| \Rightarrow \text{első főtétel:} \quad \dot{H}_2 - \dot{H}_1 = \underbrace{\dot{Q}_{12}}_{=0\ /\text{adiabatikus}/} + \dot{W}_K$$

$$\dot{H}_4 - \dot{H}_3 = \underbrace{\dot{Q}_{34}}_{=0\ /\text{adiabatikus}/} + \dot{W}_T$$

$$\eta_K = \frac{T_{2,rev} - T_1}{T_2 - T_1} \Rightarrow T_2 = T_1 + \frac{1}{\eta_K}\left(T_{2,rev} - T_1\right) = 684{,}371\ [K]$$

$$\dot{W}_K = \dot{H}_2 - \dot{H}_1 = c_p\, \dot{m}_{be}(T_2 - T_1) > 0$$

// stac. esetben: $\dot{m}_{be} = \dot{m}_{ki}$ //

$$\dot{W}_T = \dot{H}_4 - \dot{H}_3 = c_p\, \dot{m}_{ki}(T_4 - T_3) < 0$$

$$|\dot{W}_K| = |\dot{W}_T| \Rightarrow c_p\, \dot{m}\,(T_2 - T_1) = c_p\, \dot{m}\,(T_3 - T_4)$$

$$T_2 - T_1 = T_3 - T_4 \qquad (1)$$

$$\eta_T = \frac{T_3 - T_4}{T_3 - T_{4,rev}} \qquad (2)$$

$$\frac{T_{4,rev}}{T_3} = \left(\frac{p_4}{p_3}\right)^{\frac{\varkappa-1}{\varkappa}} = \left(\frac{p_1}{p_2}\right)^{\frac{\varkappa-1}{\varkappa}} = \frac{1}{\Pi^{\frac{\varkappa-1}{\varkappa}}} \qquad (3)$$

$p_1 = p_4$
$p_2 = p_3$

(1),(3) → (2)-be:

$$\eta_T = \frac{T_2 - T_1}{T_3 - T_3 \frac{1}{\Pi^{\frac{\varkappa-1}{\varkappa}}}} = \frac{T_2 - T_1}{T_3\left(1 - \frac{1}{\Pi^{\frac{\varkappa-1}{\varkappa}}}\right)} \Rightarrow$$

$$\Rightarrow T_3 = \frac{T_2 - T_1}{\eta_T\left(1 - \frac{1}{\Pi^{\frac{\varkappa-1}{\varkappa}}}\right)} = 787{,}439\ [K]$$

$$T_{4,rev} = T_3 \frac{1}{\pi^{\frac{\kappa-1}{\kappa}}} = 356{,}601 \; [K]$$

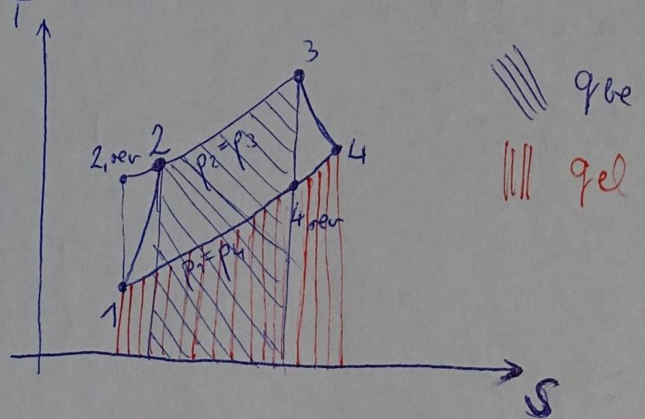
$$T_4 = T_3 - \eta_T \left(T_3 - T_{4,rev}\right) = 391{,}068 \; [K]$$

teljesítmények: (üresjáratban!)

$$|\dot{W}_K| = |\dot{W}_T| = c_p \dot{m} \left(T_2 - T_1\right) = c_p \dot{m} \left(T_3 - T_4\right) = 119{,}446 \; [MW]$$

bevezetendő hőteljesítmény:

$$\dot{Q}_{be} = c_p \dot{m} \left(T_3 - T_2\right) = 31{,}0595 \; [MW]$$




irreverzibilitások esetén: $|q_{be}| - |q_{el}| \neq |w|$ !!!

// továbbá: $\dot{Q}_{el} = c_p \dot{m} \left(T_1 - T_4\right) = -31{,}0595 \; [MW]$

$|\dot{Q}_{be}| = |\dot{Q}_{el}|$ //most//

ugyanis:

$$\dot{H}_4 - \dot{H}_1 = \dot{W}_K + \dot{Q}_{be} + \underbrace{\dot{W}_T}_{<0}$$ , továbbá $|\dot{W}_K| = |\dot{W}_T|$

$$\dot{H}_4 - \dot{H}_1 = \dot{Q}_{be}$$

ill. $\dot{H}_1 - \dot{H}_4 = c_p m \left(T_1 - T_4\right) = \dot{Q}_{el}$ //