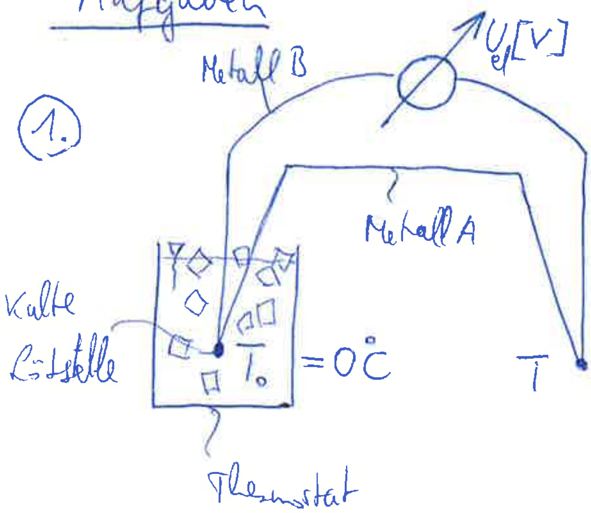
Aufgaben

(1.)



Das Thermoelement ist ein Stromkreis, ~~unterschiedliche~~ bestehend aus zwei elektrischen Leiter aus unterschiedlichem Material.
ZB: Fe-Cu, Fe-Ni, ...

Wenn die Temperaturen der kalten und warmen Lötstellen unterschiedlich sind, dann kann elektrische Spannung zwischen den kalten und warmen Lötstellen gemessen werden. Diese - sogenannte - Thermospannung ist die Funktion der Temperaturdifferenz zwischen den Lötstellen, so kann die Temperatur relativ zur kalten Lötstelle gemessen werden.

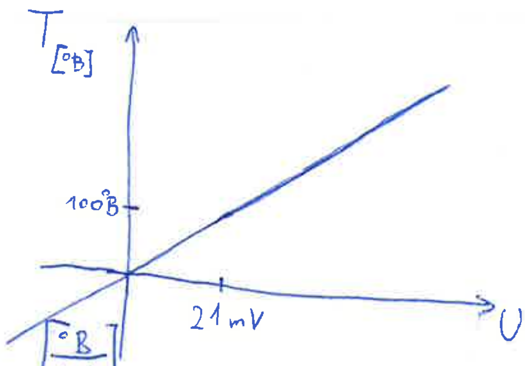
a) $U_{el} = a T_{[°C]} + b T^2_{[°C]}$, $a = 0{,}2 \frac{mV}{°C}$, $b = +10^{-4} \frac{mV}{(°C)^2}$

T [°C]	-100	0	100	200	300	400	500
U_{el} [mV]	-19	0	21	44	69	96	125

b) $T_{[°B=0]} = T_{[°C=0]} \mathrel{\hat{=}} 0 \text{ mV}$

$T_{[°B=100]} = T_{[°C=100]} \mathrel{\hat{=}} 21 \text{ mV}$

$T_{[°B]} = c\, U_{el} \Rightarrow c = \frac{T_{[°B=100]}}{U_{el}[B=100]} = \frac{100}{21} \left[\frac{°B}{mV}\right]$



T [°C]	-100	0	100	200	300	400	500
U [mV]	-19	0	21	44	69	96	125
T [°B]	-90,48	0	100	209,52	328,57	457,14	595,24

$$U(T_{[°B]}) = U(T_{[°C]})$$

$$T_{[°B]} = a_C T_{[°C]} + b_C T_{[°C]}^2 = \frac{20}{21} T_{[°C]} + \frac{1}{2100} T_{[°C]}^2$$

(2) $T_{[K]} = T_{[°C]} + 273,15 = 490,15 \text{ K}$

(3) $T_{[°F]} = 32 + \frac{9}{5} T_{[°C]} = 68 °F$

$$T_{[°R]} = \frac{9}{5} T_{[K]} = \frac{9}{5}\left(T_{[°C]} + 273,15\right) = \frac{9}{5} T_{[°C]} + 491,67 = 527,67$$

(4) $V = \text{konst}$

$p_2 = 1,4382\, p_1$ $\qquad T_1 = 0 °C$

$$pV = mRT \rightsquigarrow pv = RT \rightsquigarrow \frac{p}{T} = \frac{R}{v} = \text{konst}$$

$$\frac{p_2}{T_2} = \frac{p_1}{T_1} \Rightarrow T_2 = \frac{p_2}{p_1} T_1 = 392,844 \text{ K} = 119,694 °C$$

(5) $v = \text{konst}$

$p_2 = 1,36605\, p_1$ $\qquad T_1 = 273,16 \text{ K}$

$$pv = RT \rightsquigarrow \frac{T}{p} = \frac{v}{R} = \text{konst} \Rightarrow T_2 = \frac{p_2}{p_1} T_1 = 373,15 \text{ K} = 100 °C$$

$\cancel{99,9866 °C \approx 100 °C}$

(6) $p_1 = 1,333$ bar $\qquad T_1 = 0 °C$

$p_2 = 1,821$ bar $\qquad T_2 = 100 °C$

a) $T(p) = ap + b$

$$T(p_1) = T_1 = a p_1 + b \qquad (1)$$

$$T(p_2) = T_2 = a p_2 + b \qquad (2)$$

(2) − (1) :

$$T_2 - T_1 = a(p_2 - p_1) \Rightarrow a = \frac{T_2 - T_1}{p_2 - p_1} = 204{,}918 \ \frac{K}{bar}$$

$(2)p_1 - (1)p_2$:

$$T_2 p_1 - T_1 p_2 = b(p_1 - p_2) \Rightarrow b = \frac{T_2 p_1 - T_1 p_2}{p_1 - p_2} =$$

$$= \frac{(T_{2[°C]} - 273{,}15)p_1 - (T_{1[°C]} - 273{,}15)p_2}{p_1 - p_2} = \frac{T_{2[°C]}p_1 - T_{1[°C]}p_2}{p_1 - p_2} - 273{,}15$$

$$= -273{,}15574 \ °C = -0{,}00574 \ K$$

b) $T(p) = ap + b \Rightarrow T(1{,}431 \ bar) = 293{,}232 \ K = 20{,}082 \ °C$

7 $V = 3 \ dm^3$ $\quad \mathcal{R} = 8314 \ \frac{J}{kmol \, K}$ $\quad M = 39{,}95 \ \frac{kg}{kmol}$

$m_1 = 10 \ g$ $\quad p_1 = 2{,}072 \ bar$ $\quad R = \frac{\mathcal{R}}{M}$

$m_2 = 3 \ g$ $\quad p_2 = 0{,}4159 \ bar$

$$\frac{pv}{RT} = 1 + B(T)p$$

$$\frac{p_1 V}{m_1 R T} = 1 + B(T)p_1 \qquad (1)$$

$$\frac{p_2 V}{m_2 R T} = 1 + B(T)p_2 \qquad (2)$$

$(1)p_2 - (2)p_1$

$$\frac{p_1 p_2 V}{m_1 RT} - \frac{p_1 p_2 V}{m_2 RT} = p_2 - p_1$$

$$\frac{p_1 p_2 V(m_2 - m_1)}{m_1 m_2 RT} = p_2 - p_1 \Rightarrow T = \frac{p_1 p_2 V(m_2 - m_1)}{m_1 m_2 \frac{R}{M}(p_2 - p_1)} = T = 175{,}024\,K$$

$(1) - (2)$

$$\frac{p_1 V}{m_1 RT} - \frac{p_2 V}{m_2 RT} = B(T)(p_1 - p_2) \Rightarrow B(T) = \frac{V}{RT}\left[\frac{p_1 m_2 - p_2 m_1}{(m_1 m_2)(p_1 - p_2)}\right] =$$

$$= 3{,}41 \cdot 10^{-6} \left[\frac{1}{Pa}\right]$$

(8) $T_{E_0} = -5^\circ C$ $\quad m_{E_0} = 1kg$ $\quad c_E = 2{,}01 \frac{kJ}{kg\,K}$ $\quad q_{sch} = 335 \frac{kJ}{kg}$

$T_{W_0} = 20^\circ C$ $\quad m_{W_0} = 2kg$ $\quad c_W = 4{,}187 \frac{kJ}{kg\,K}$

$T_{E_0} \rightarrow 0^\circ C$

$$Q_{(-5\to 0^\circ C)} = c_E m_{E_0} (\underset{[0^\circ C]}{T} - T_{E_0}) = 10{,}05\ kJ$$

$T_{W_0} \rightarrow 0^\circ C$

$$Q_{(20\to 0^\circ C)} = c_W m_W (T_{[0^\circ C]} - T_{W_0}) = -167{,}48\ kJ$$

Schmelzen:

$$Q_{Schm} = m_{E_0} q_{sch} = 335\ kJ < |Q_{(20\to 0^\circ C)}| - |Q_{(-5\to 0^\circ C)}|$$

$$m_{E,Schm} = \frac{Q_{(20\to 0^\circ C)} - Q_{(-5\to 0^\circ C)}}{q_{sch}} = 0{,}46994$$

$$m_{E,gl} = m_{E_0} - m_{E,Sch} = 0,53 \text{ kg}$$

$$m_{W,gl} = m_{W_0} + m_{E,Sch} = 2,47 \text{ kg}$$

9 Beweis: $\dfrac{\partial\beta}{\partial p} = -\dfrac{\partial\chi}{\partial T}$

$$\beta(p,T) = \frac{1}{v}\left.\frac{\partial v}{\partial T}\right|_p \qquad \chi(p,T) = -\frac{1}{v}\left.\frac{\partial v}{\partial p}\right|_T$$

$$\frac{\partial\beta}{\partial p} = \frac{\partial}{\partial p}\left(\frac{1}{v}\right)\left.\frac{\partial v}{\partial T}\right|_p + \frac{1}{v}\frac{\partial^2 v}{\partial T\partial p} = -\frac{1}{v^2}\left.\frac{\partial v}{\partial p}\right|_T\left.\frac{\partial v}{\partial T}\right|_p + \frac{1}{v}\frac{\partial^2 v}{\partial T\partial p}$$

$$\frac{\partial\chi}{\partial T} = -\frac{\partial}{\partial T}\left(\frac{1}{v}\right)\left.\frac{\partial v}{\partial p}\right|_T - \frac{1}{v}\frac{\partial^2 v}{\partial p\partial T} = \frac{1}{v^2}\left.\frac{\partial v}{\partial T}\right|_p\left.\frac{\partial v}{\partial p}\right|_T - \frac{1}{v}\frac{\partial^2 v}{\partial p\partial T}$$

Satz von Schwartz (- Young): $\dfrac{\partial^2 v}{\partial T\partial p} = \dfrac{\partial^2 v}{\partial p\partial T}$

$$\Rightarrow \frac{\partial\beta}{\partial p} = -\frac{\partial\chi}{\partial T}$$

$\square$

10 H_2O: $M = 18 \frac{\text{kg}}{\text{kmol}}$ $\quad T = 300°C = 573\text{ K}$

p [bar]	75	25	5	1	0,1	0,02
$\frac{pV_{mol}}{T}$ $\left[\frac{J}{mol\,K}\right]$	6314,4	7774,87	8183,25	8261,78	8293,19	8293,19

CO_2: $M = 44 \frac{\text{kg}}{\text{kmol}}$ $\quad T = 47°C = 320\text{ K}$

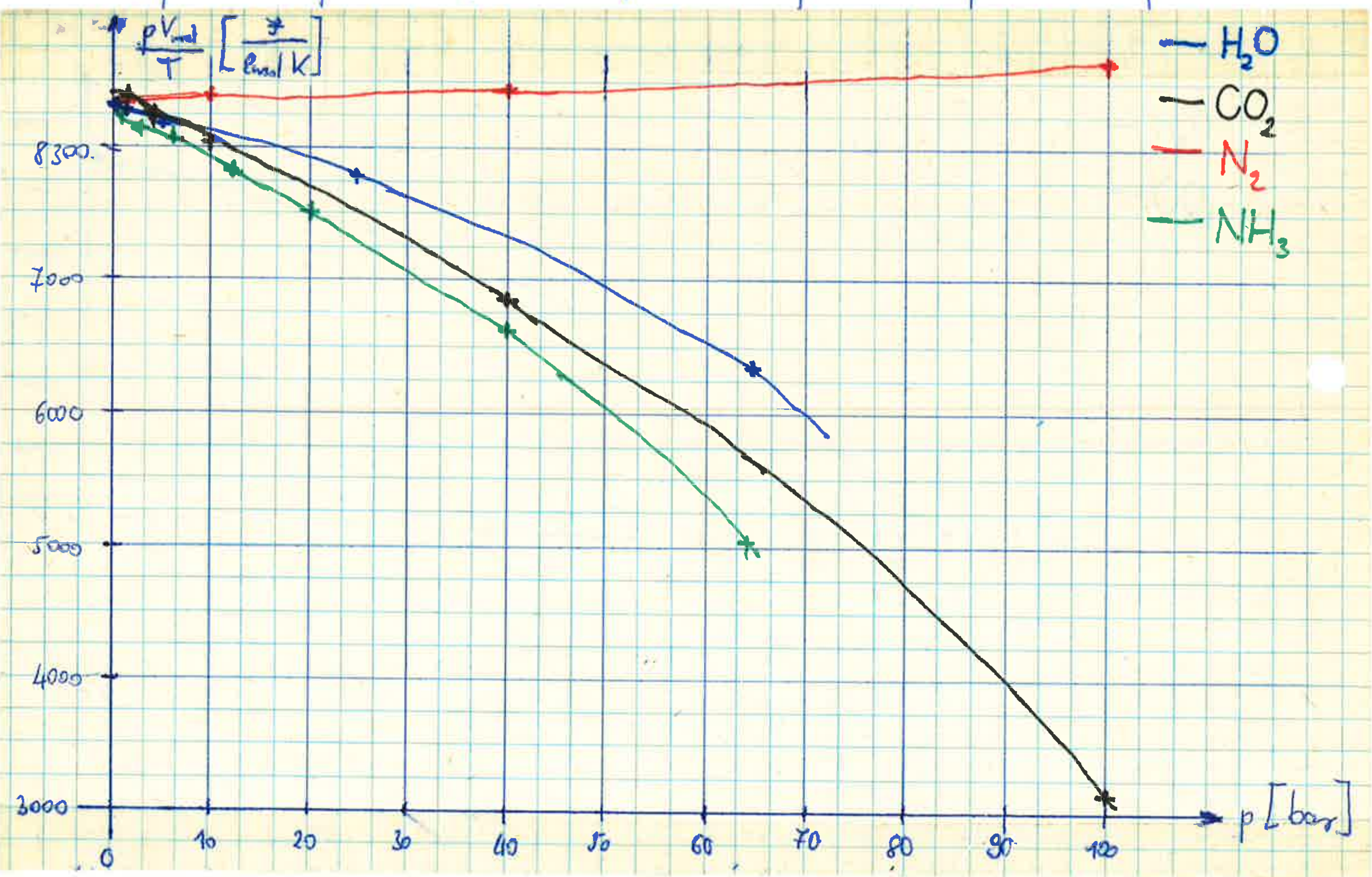
p [bar]	100	40	10	4	1	0,01
$\frac{pV_{mol}}{T}$ $\left[\frac{J}{mol\,K}\right]$	3121,25	6847,5	8016,25	8250	8318,75	8373,75

N_2: $M = 28 \frac{g}{mol}$ $T = 427\,°C = 700\,K$

p [bar]	100	40	10	1	0,01
$\frac{pV_{mol}}{T}$ $\left[\frac{J}{mol\,K}\right]$	8680	8448	8360	8320	8320

NH_3: $M = 17 \frac{g}{mol}$ $T = 100\,°C = 373\,K$

p [bar]	64	40	20	12	8	4	1
$\frac{pV_{mol}}{T}$ $\left[\frac{J}{mol\,K}\right]$	5031,64	6617,69	7501,88	7820,91	8057,91	8130,83	8294,91



für „~~kleine~~" ~~Drücke~~ $p \cong 0$ $\frac{pV_{mol}}{T} \approx 1$, aber für „kleine" ~~große~~ Drücke $\frac{pV_{mol}}{T} = f(T) \Rightarrow$

$$\frac{pV_{mol}}{RT} = 1 + f(T)\,p$$

(11) überhitzten Ammoniakdampf: NH_3 $\qquad M = 17 \frac{kg}{kmol}$

$$v(p,T) = \frac{RT}{p} - a\left(\frac{d}{T}\right)^3 - (b + cp^2)\left(\frac{d}{T}\right)^{11} \underset{\substack{p=2bar \\ T=30°C}}{=} 0{,}7256 \left[\frac{m^3}{kg}\right]$$

$a = 0{,}42 \frac{m^3}{kg} \qquad b = 60 \frac{m^3}{kg} \qquad d = 100\,K \qquad c = 1{,}497 \cdot 10^{-11} \frac{m^7}{kg\,N^2}$

$$\beta(p,T) = \frac{1}{v}\frac{\partial v}{\partial T}\Big|_p \qquad \chi(p,T) = -\frac{1}{v}\frac{\partial v}{\partial p}\Big|_T$$

$$\frac{\partial v}{\partial T} = \frac{R}{p} + \frac{3ad^3}{T^4} + \frac{11(b+cp^2)d^{11}}{T^{12}} \underset{\substack{p=2bar \\ T=30°C}}{=} 0{,}0026 \frac{m^3}{kg\,K}$$

$$\frac{\partial v}{\partial p} = -\frac{RT}{p^2} - 2cp\left(\frac{d}{T}\right)^{11} \underset{\substack{p=2bar \\ T=30°C}}{=} -3{,}7048 \cdot 10^{-6} \left[\frac{m^5}{kg\,N} = \frac{m^3}{kg\,Pa}\right]$$

$$\beta = 0{,}003592 \frac{1}{K} \qquad \chi = 5{,}1062 \cdot 10^{-6} \left[\frac{m^2}{N} = \frac{1}{Pa}\right]$$

(12) $\beta = \frac{R}{pv} \qquad \chi = \frac{1}{p}$

$$\beta = \frac{1}{v}\frac{\partial v}{\partial T}\Big|_p = \frac{R}{pv} \Rightarrow \frac{\partial v}{\partial T}\Big|_p = \frac{R}{p}$$

$$\chi = -\frac{1}{v}\frac{\partial v}{\partial p}\Big|_T = \frac{1}{p} \Rightarrow \frac{\partial v}{\partial p}\Big|_T = -\frac{v}{p}$$

$$v(p,T) = \int \frac{\partial v}{\partial T}\Big|_p dT + f(p) = \frac{RT}{p} + f(p)$$ ⇝ isobare Linien

$$\cancel{\chi(p,T)} = -\frac{1}{v}\frac{\partial v}{\partial p}\Big|_T = \frac{1}{p}$$

$$\frac{dv}{v}\Big|_T = -\frac{dp}{p} \qquad /\int$$

$$\ln v = -\ln p + \ln g(T)$$

$$\ln(pv) = \ln g(T)$$

$$pv = g(T) \quad \rightsquigarrow \text{ isochore Linien}$$

$$v(p,T) = \frac{g(T)}{p} = \frac{RT}{p} + f(p)$$

$$g(T) = RT \qquad f(p) = 0$$

$$\boxed{pv = RT}$$

13 $\beta = \frac{1}{v}\frac{\partial v}{\partial T}\Big|_p = \frac{R}{pv}$ $\qquad \chi = -\frac{1}{v}\frac{\partial v}{\partial p}\Big|_T = \frac{1}{p} + \frac{a}{v}$

$$v(p,T) = \int \frac{\partial v}{\partial T}\Big|_p dT + f(p) =$$

$$= \frac{RT}{p} + f(p)$$

$$\frac{dv}{v}\Big|_T + \frac{dp}{p}\Big|_T = -\frac{a}{v}\,dp\Big|_T$$

$$p\,dv|_T + v\,dp|_T = -a\,p\,dp|_T \quad /\int$$

$$pv = \int d(pv)\Big|_T = -a\int p\,dp\Big|_T = -\frac{ap^2}{2} + g(T)$$

$$v(p,T) = \frac{RT}{p} + f(p) = -\frac{ap}{2} + \frac{g(T)}{p}$$

$$g(T) = RT$$

$$f(p) = -\frac{ap}{2}$$

$$\boxed{pv = RT - \frac{ap^2}{2}}$$

(14)

$p_1 = 10 \text{ bar}$ $\quad T_1 = 100\,°C$ $\quad v = \text{konst}$

$p_2 = ?$ $\quad T_2 = 104\,°C$

$\beta = 0{,}00268 \frac{1}{K}$ $\quad \chi = 10^{-10} \frac{m^2}{N}$

$$p = p(v,T)$$

$$dp = \frac{\partial p}{\partial v}\Big|_T dv + \frac{\partial p}{\partial T}\Big|_v dT \overset{\text{isochore Zä.}}{\doteq} \frac{\partial p}{\partial T}\Big|_v dT$$

$$v = v(p,T)$$

$$\underbrace{dv}_{=0} = \frac{\partial v}{\partial p}\Big|_T dp + \frac{\partial v}{\partial T}\Big|_p dT = \frac{\partial v}{\partial p}\Big|_T \frac{\partial p}{\partial v}\Big|_T \underbrace{dv}_{=0 \text{ (isochore Zä.)}} + \left(\frac{\partial v}{\partial p}\Big|_T \frac{\partial p}{\partial T}\Big|_v + \frac{\partial v}{\partial T}\Big|_p\right) dT$$

$$0 = \left(\frac{\partial v}{\partial p}\Big|_T \frac{\partial p}{\partial T}\Big|_v + \frac{\partial v}{\partial T}\Big|_p\right) dT \qquad \forall dT$$

$$-\frac{\partial v}{\partial T}\Big|_p = \frac{\partial v}{\partial p}\Big|_T \frac{\partial p}{\partial T}\Big|_v \Longrightarrow \frac{\partial p}{\partial T}\Big|_v = + \frac{\frac{1}{v}\frac{\partial v}{\partial T}\Big|_p}{-\frac{1}{v}\frac{\partial v}{\partial p}\Big|_T} = \frac{\beta}{\chi}$$

$$\boxed{-1 = \frac{\partial v}{\partial p}\Big|_T \frac{\partial p}{\partial T}\Big|_v \frac{\partial T}{\partial v}\Big|_p}$$

$$p_2 = p_1 + \underbrace{\frac{\beta}{\chi}(T_2 - T_1)}_{\frac{\partial p}{\partial T}\big|_v dT} = 1{,}073 \cdot 10^8 \text{ Pa} \triangleq 1073 \text{ bar}$$

große Änderung im Druck verursacht von kleiner Änderung in Temperatur $\Rightarrow$ Flüssigkeit

(15)

$V_1 = 10 \text{ dm}^3$ $\quad T_1 = T_2 = 20\,°C$

$p_1 = 1 \text{ bar}$ $\quad p_2 = 3000 \text{ bar}$

$\chi = a + b\,p$ $\quad a = 3{,}9 \cdot 10^{-11} \frac{m^2}{N}$ $\quad b = -1{,}7 \cdot 10^{-20} \frac{m^4}{N^2}$

$$\chi = -\frac{1}{V}\frac{\partial V}{\partial p}\bigg|_T = a + b_p$$

$$-\frac{dV}{V}\bigg|_T = (a + b_p)\,dp\bigg|_T$$

$$-\ln\frac{V}{V_1} = a(p - p_1) + \frac{b}{2}(p^2 - p_1^2)$$

$$V(p) = V_1\, e^{-a(p-p_1) - \frac{b}{2}(p^2 - p_1^2)}$$

$$V_2 = 9{,}8913 \ dm^3$$

$$l_{12} = -\int_{V_1}^{V_2} p(V)\,dV = -\int_{V_1}^{V_2} -\frac{a}{b} + \frac{1}{b}\sqrt{c - 2b\ln\frac{V}{V_1}}\,dV = \ldots = 15903{,}1 \ J$$

erf oder numerische Integration

$$\frac{b}{2}p^2 + ap + \left[-\frac{b}{2}p_1^2 - ap_1 + \ln\frac{V}{V_1}\right] = 0$$

$$p_{1,2} = \frac{-a \boxed{+} \sqrt{a^2 - 2b\left(-\frac{b}{2}p_1^2 - ap_1 + \ln\frac{V}{V_1}\right)}}{b} =$$

$$= -\frac{a}{b} + \frac{1}{b}\sqrt{\underbrace{a^2 + b^2p_1^2 + 2abp_1}_{=:c \,=\, 1{,}52087\cdot 10^{-21}\ \frac{m^4}{N^2}} - 2b\ln\frac{V}{V_1}} =$$

$$\hat{=} -\frac{a}{b} + \frac{1}{b}\sqrt{c - 2b\ln\frac{V}{V_1}}$$

(16) $m = 5\,kg$ $\quad p = konst$ $\quad T_1 = 200\,°C$ $\quad T_2 = 600\,°C$

$$\bar{c}_p\Big|_0^{200} = 0{,}935\ \frac{kJ}{kg\,K} \qquad \bar{c}_p\Big|_0^{600} = 0{,}993\ \frac{kJ}{kg\,K}$$

$$q\Big|_0^{600} = q\Big|_0^{200} + q\Big|_{200}^{600} \Rightarrow q\Big|_{200}^{600} = q\Big|_0^{600} - q\Big|_0^{200} =$$

$$= \bar{c}_p\Big|_0^{600}(T_2 - T_{[0°C]}) - \bar{c}_p\Big|_0^{200}(T_1 - T_{[0°C]}) = 408\,800\ \frac{J}{kg}$$

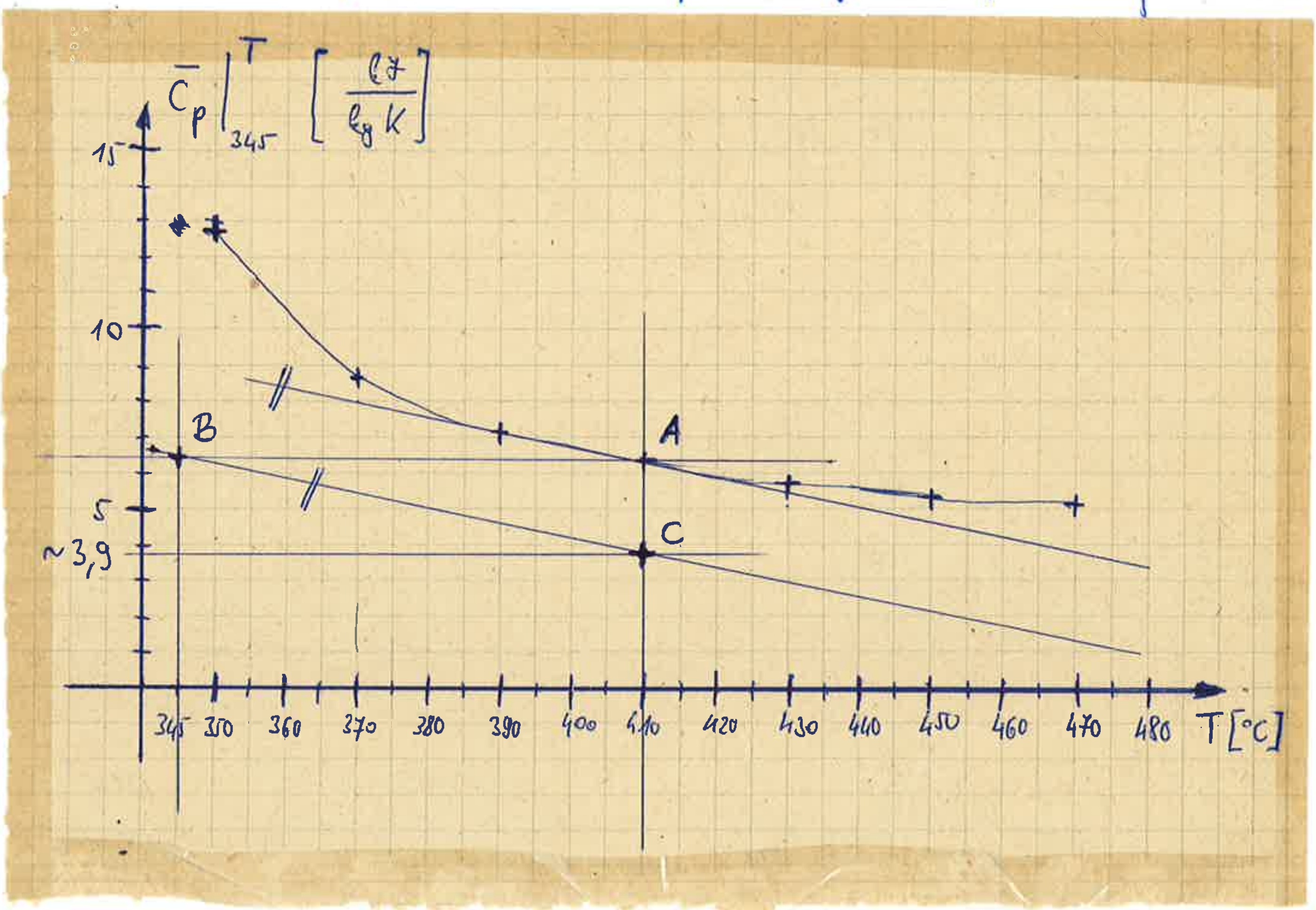
$$Q\Big|_{200}^{600} = m\,q\Big|_{200}^{600} = 2044\ kJ$$

durchschnittliche spezifische isobare Wärmekapazität:

$$q\Big|_{200}^{600} = \bar{c}_p\Big|_{200}^{600} \, \Delta T\Big|_{200}^{600} \Rightarrow \bar{c}_p\Big|_{200}^{600} = \frac{q\big|_{200}^{600}}{\Delta T\big|_{200}^{600}} = 1{,}022 \; \frac{kJ}{kg\,K}$$

(17)

$T\,[°C]$	350	370	390	410	430	450	470
$\bar{c}_p\Big\vert_{345}^{T}\left[\frac{kJ}{kg\,K}\right]$	12,96	8,70	7,15	6,36	5,82	5,36	5,02



Graphische Lösung: 1) $\bar{c}_p\Big|_{345}^{T}(T)$ Funktion

2) $\bar{c}_p\Big|_{345}^{410} \rightsquigarrow A$ projizieren auf $T = 345\,°C \rightsquigarrow B$

3) Tangente von $\bar{c}_p\Big|_{345}^{T}(T = 410) \rightsquigarrow$ parallel Gerade durch B

4) $T = 410\,°C \cap$ parallel durch $B \rightsquigarrow C$

5) projizieren C auf $\bar{c}_p\Big|_{345}^{T} \rightsquigarrow$ wahre spezifische Wärmekapazität

Numerische Lösung:

$$c_p(T_1) = \bar{c}_p\Big|_{T_0}^{T_1} + \frac{d\bar{c}_p\Big|_{T_0}^{T}}{dT}(T_1)\,(T_1 - T_0) \approx$$

$$\approx \bar{c}_p\Big|_{T_0}^{T_1} + \frac{\bar{c}_p\Big|_{T_0}^{T+\Delta T} - \bar{c}_p\Big|_{T_0}^{T-\Delta T}}{2\Delta T \;\; (T+\Delta T-(T-\Delta T))}(T_1 - T_0) =$$

$$= \bar{c}_p\Big|_{345}^{410} + \frac{\bar{c}_p\Big|_{345}^{430} - \bar{c}_p\Big|_{345}^{390}}{40}(410-345) = 4{,}19875\left[\frac{kJ}{kg\,K}\right]$$

(18) $m = 0{,}05\ kg$ $\quad T_0 = 20\,°C$

$c = 1{,}76\ \frac{kJ}{kg\,K}$ $\quad T_1 = 400\,°C$

$Q_{nötig} = c\,m\,(T_1 - T_0) = 33{,}44\ kJ \Rightarrow W = 2Q = 66{,}88\ kJ$

(19) $\Delta U = 12\ V$ $\quad R = 3\,\Omega$ $\quad T = konst$ $\quad t = 1h$

$P = UI = \frac{U^2}{R} = I^2 R = 48\ W = 48\ \frac{J}{s}$

$Q = Pt = 172{,}8\ kJ$

(20) $p = 8\ bar = konst$

$T_1 = 30\,°C$ $\quad T_2 = 400\,°C$

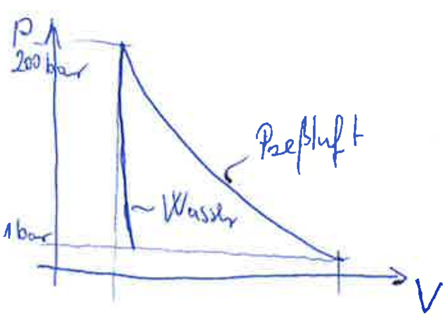
$V_1 = 10\ dm^3$ $\quad V_2 = 22{,}41\ dm^3$

$$l_{12} = -\int_{V_1}^{V_2} p(V)\,dV = -p(V_2 - V_1) = -9928\ J$$

falls $V = konst$

$$l_{12} = -\int_{V_1}^{V_2 = V_1} p(V)\,dV = 0$$

(21)



$l_{12,W} \ll l_{12,Pl}$

$\downarrow$

besser für Druckprobe

(22) a) $p\uparrow$ b) $p\downarrow$ c) $p\uparrow$

(23) a) $T\uparrow$ $l_{12}\uparrow$ b) $l_{12}\uparrow$ $q_{12}\downarrow$

c) $pV = mRT$ $\quad \frac{V}{T} = \frac{mR}{p} = konst$ $\quad \frac{V_1}{T_1} = \frac{V_2}{T_2} = \frac{2V_1}{T_2} \Rightarrow T_2 = 2T_1$

(24) $\dot{m} = 100\,\frac{t}{h}$ $\quad p = 100$ bar $\quad v = 33{,}52\,\frac{dm^3}{kg}$ $\quad T = 500\,°C$

$$\dot{L}_V = +p v \dot{m} = 9{,}31\,\frac{MJ}{s} = 9{,}31\text{ MW}$$

$[Pa]\left[\frac{m^3}{kg}\right]\left[\frac{kg}{s}\right]$

(25) $\dot{m} = 800\,\frac{kg}{h}$ $\quad T = 400\,°C$

$p = 1$ bar

$v = 1{,}97\,\frac{m^3}{kg}$

$$\dot{L}_V = -p v \dot{m} = -43{,}7778\,\frac{kJ}{s} = -43{,}78\text{ kW}$$

Energie des Systems fällt ab

(26) $m = 6\,kg$

$Q_{12} = 100\,kJ$

$L_{12} = -50\,kJ$

$$U_2 - U_1 = Q_{12} + L_{12} = 50\,kJ$$

$$u_2 - u_1 = \frac{1}{m}(U_2 - U_1) = 8{,}33\,\frac{kJ}{kg}$$

(27) $Q_{12} = 0$

$L_{12} = 90\,kJ$

$$U_2 - U_1 = Q_{12} + L_{12} = 90\,kJ$$

(28) $Q_{12} = 80\,kJ$

$V = konst$

$$U_2 - U_1 = Q_{12} + L_{12} = Q_{12} + \underbrace{\int_{V_1}^{V_2=V_1} p(V)\,dV}_{=0}$$

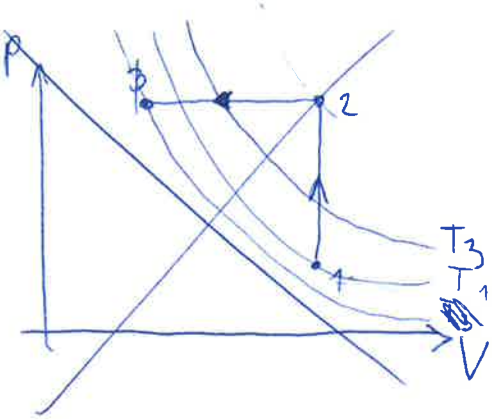
Wärme erhöht sich die innere Energie

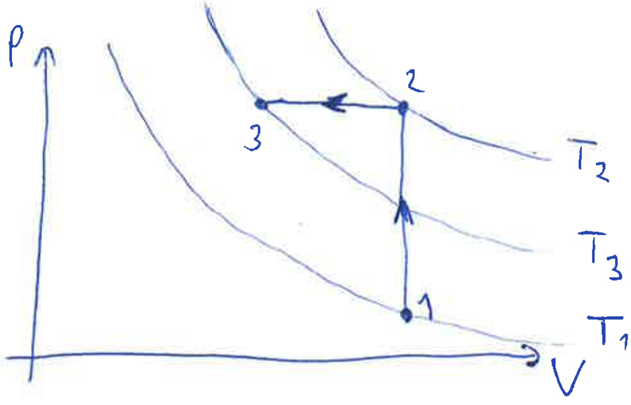
(29) $m = 3\,kg$

$p_1 = 1$ bar $\quad p_3 = 2{,}5$ bar $\quad R = 287\,\frac{J}{kg\,K}$

$T_1 = 300\,K$ $\quad T_3 = 400\,K$ $\quad c_v = 717\,\frac{J}{kg\,K}$

$$c_p = R + c_v = 1004\,\frac{J}{kg\,K}$$





$$V_1 = V_2 = \frac{m R T_1}{p_1} = 2{,}583\ m^3$$

$$V_3 = \frac{m R T_3}{p_3} = 1{,}3776\ m^3$$

$$\frac{pV}{T} = konst$$

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2} \Rightarrow T_2 = T_1 \frac{p_2 V_2}{p_1 V_1} =$$

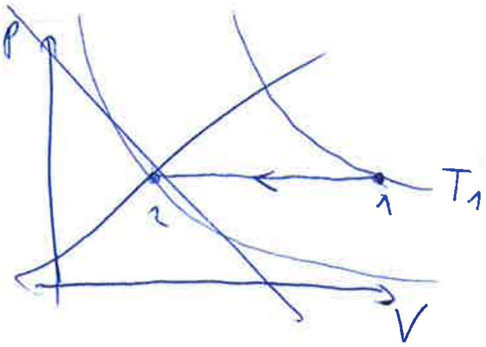
$$= T_1 \frac{p_3}{p_1} \underbrace{\frac{V_1}{V_1}}_{=1} = 750\ K$$

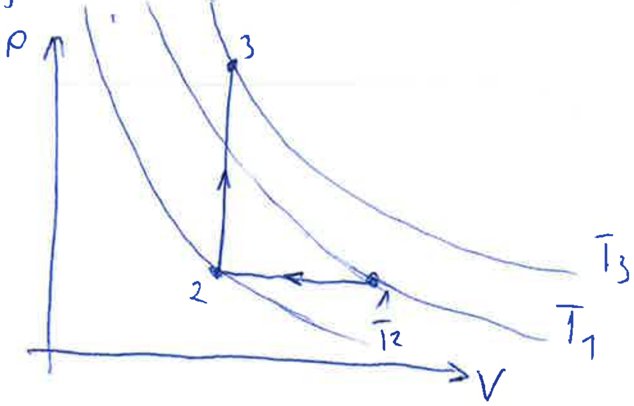
$$L_{13} = L_{12} + L_{23} = \underbrace{-\int_{V_1}^{V_2} p(V)\,dV}_{=0} - \int_{V_2}^{V_3} p(V)\,dV = -p_3 (V_3 - V_2) = 301{,}35\ kJ$$

$$Q_{13} = Q_{12} + Q_{23} = m c_v (T_2 - T_1) + m c_p (T_3 - T_2) = -86{,}25\ kJ$$

$$U_3 - U_1 = m c_v (T_3 - T_1) = 215{,}1\ kJ$$

$$U_3 - U_1 = Q_{13} + L_{13} = 215{,}1\ kJ$$





$$L_{13} = L_{12} + L_{23} = -p_1 (V_3 - V_1) + 0 = 120{,}540\ kJ$$

$$Q_{13} = Q_{12} + Q_{13} = m c_p (T_2 - T_1) + m c_v (T_3 - T_2) = 94{,}56\ kJ$$

$$T_2 = T_1 \underbrace{\frac{p_2}{p_1}}_{=1} \frac{V_2}{V_1} = T_1 \frac{V_3}{V_1} = 160\ K$$

$$U_3 - U_1 = Q_{13} + L_{13} = 215{,}1\ kJ$$

(30) Ar:

$M = 40 \frac{kg}{kmol}$ $\qquad \kappa = 1{,}67$

$\mathcal{R} = 8314{,}4 \frac{J}{kmol\,K}$

$$R = \frac{\mathcal{R}}{M} = 207{,}86 \frac{J}{kg\,K}$$

$$\begin{cases} c_p - c_v = R \Rightarrow (\kappa-1)c_v = R \\ \frac{c_p}{c_v} = \kappa \Rightarrow c_p = \kappa c_v \end{cases}$$

$$\Rightarrow c_v = \frac{R}{\kappa-1} = 310{,}239 \frac{J}{kg\,K}$$

$$\Downarrow$$

$$c_p = \kappa c_v = 518{,}099 \frac{J}{kg\,K}$$

$$c_p = \frac{\kappa}{\kappa-1} R$$

(31) $T \in [-100°C, 100°C]$ $\qquad u(T_0 = 0°C) = 0 \frac{kJ}{kg}$

$c_p = 1{,}0 \frac{kJ}{kg\,K}$ $\qquad \kappa = 1{,}4$

$$c_v = \frac{c_p}{\kappa} = 0{,}714 \frac{kJ}{kg\,K}$$

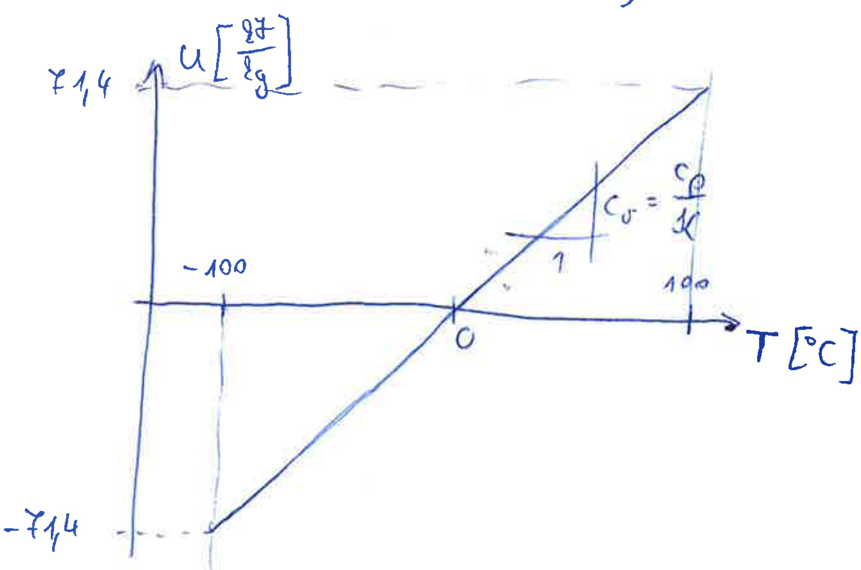
$$du = c_v(T)\,dT \underset{PG}{=} c_v\,dT$$

$$u(T) = c_v T + konst$$

$$u(T_0) = c_v T_0 + konst = 0 \Rightarrow konst = -c_v T_0$$

$$u(T) = c_v(T - T_0) = c_v\left[\left(T_{[°C]} + 273{,}15\right) - \left(T_{0[°C]} + 273{,}15\right)\right] =$$

$$= c_v\left(T_{[°C]} - T_{0[°C]}\right) = c_v T_{[°C]}$$



(32) $0°C < T < 600°C$ $\quad CO_2$

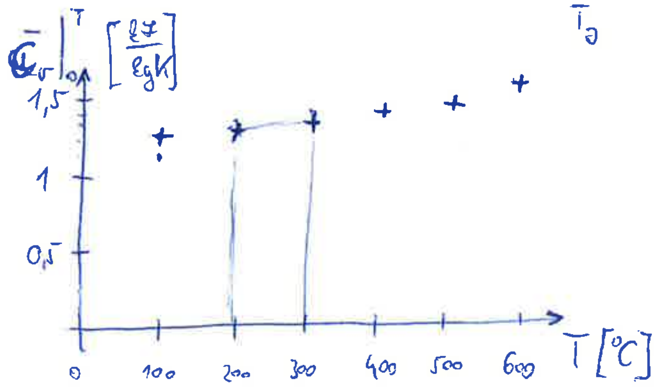
$u\,(T = 0°C) = 0\ \frac{kJ}{kg}$

T [°C]	100	200	300	400	500	600
$\bar{c}_v\big\vert_0^T \left[\frac{kJ}{kg\,K}\right]$	1,29	1,31	1,34	1,38	1,43	1,49
$u \left[\frac{kJ}{kg}\right]$	129	262	402	552	715	894

$c_v = c_v(T)$ $\quad du = c_v(T)\,dT$

$$u(T) = \int_{T_0}^{T} c_v(T)\,dT \cong \cancel{\sum_{i=0} \frac{\bar{c}_v\big|_0^{T_i} + \bar{c}_v\big|_0^{T_{i+1}}}{2}\,(T_{i+1} - T_i)}$$

$$\cong \bar{c}_v\big|_0^{T}\,(T - T_0)$$



(33)

$v \left[\frac{m^3}{kg}\right]$	0,02	0,04	0,08
$\frac{\partial u}{\partial v}\Big\vert_{T=350°C} \left[\frac{kJ}{m^3}\right]$	9180	2982	417

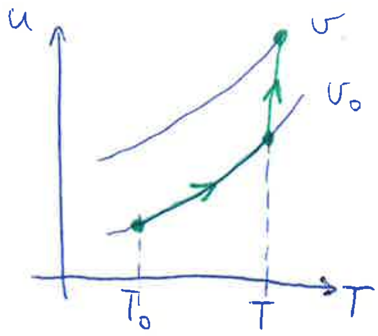
$c_v\big|_{350°C}^{T} \left[\frac{kJ}{kg\,K}\right]$:

$v \left[\frac{m^3}{kg}\right]$ \ T [°C] $\quad c_v\big\vert_{350}^{T}$	350	400	450	500	550
0,02	2,5	2,17	1,98	1,94	1,92
0,04	2,09	1,96	1,88	1,84	1,84
0,08	1,88	1,83	1,8	1,8	1,8

$$u = u(v, T)$$

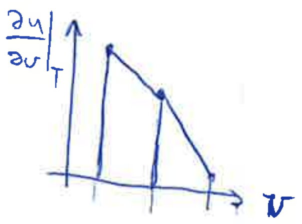
$$du = \frac{\partial u}{\partial v}\Big|_T dv + \frac{\partial u}{\partial T}\Big|_v dT = \frac{\partial u}{\partial v}\Big|_T dv + c_v\,dT$$

$$u = u(v,T) = \int_{v_0}^{v} \frac{\partial u}{\partial \tilde{v}}\Big|_{T=350°C} d\tilde{v} + \int_{T_0}^{T} c_v\Big|_{\tilde{v}=v} d\tilde{T}$$



Numerische integration (Trapezregel)

$$\int_{v_0}^{v} \frac{\partial u}{\partial \tilde{v}}\Big|_{T=350°C} d\tilde{v} = \frac{1}{2}\left[\frac{\partial u}{\partial v}\Big|_T (v_1) + \frac{\partial u}{\partial v}\Big|_T (v_0)\right](v_1 - v_0) +$$



$$+ \frac{1}{2}\left[\frac{\partial u}{\partial v}\Big|_T (v_2) + \frac{\partial u}{\partial v}\Big|_T (v_1)\right](v_2 - v_1) \Big(+$$

$$\Big(+ \frac{1}{2}\left[\frac{\partial u}{\partial v}\Big|_T (v_0) + \frac{\partial u}{\partial v}\Big|_T (v_0)\right](v_0 - v_0)\Big) =$$

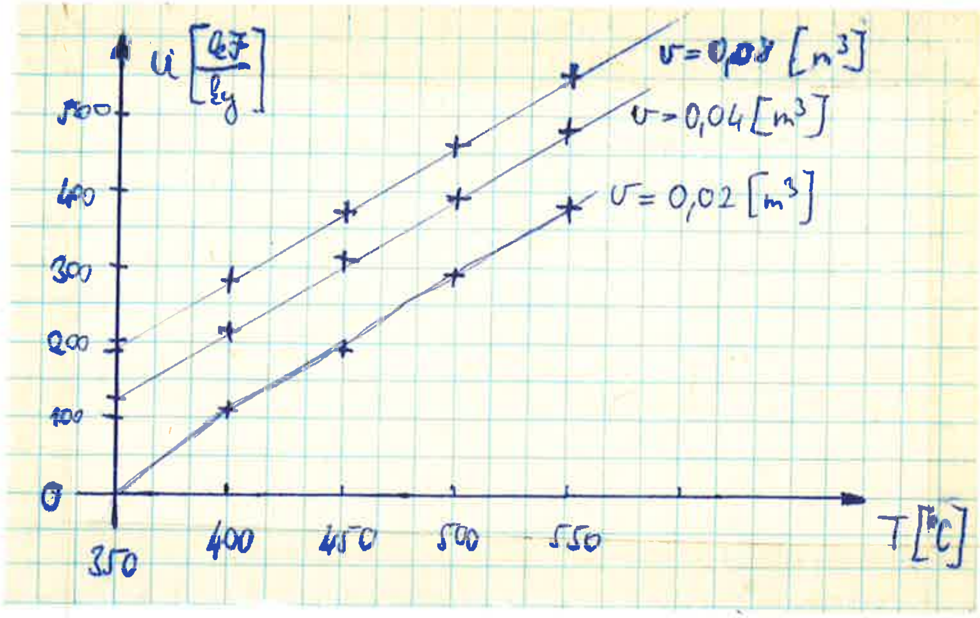
$$= 0 + 121{,}62 + 67{,}98 = 189{,}6 \ \frac{kJ}{kg}$$

$$\int_{T_0}^{T} \bar{c}_v \Big|_{T_0}^{T} d\tilde{T} = \bar{c}_v\Big|_{T_0}^{T}(T - T_0):$$

$v\left[\frac{m^3}{kg}\right]$ \ T [°C]	350	400	450	500	550
0,02	0	108,5	198	291	384
0,04	0	98	188	276	368
0,08	0	91,5	180	270	360

$u(v,T)\left[\frac{kJ}{kg}\right]$:

$v\left[\frac{m^3}{kg}\right]$ \ T [°C]	350	400	450	500	550
0,02	0	108,5	198	291	384
0,04	121,62	219,62	309,62	397,62	489,62
0,08	189,6	281,1	369,6	459,6	549,6



(36) $p_1 = 100$ bar $\qquad p_2 = 70$ bar

$T_1 = 30\,°C \qquad T_2 = 1\,°C$

$$\frac{T_2}{T_1} = \left(\frac{p_1}{p_2}\right)^{\frac{1-\varkappa}{\varkappa}} \Rightarrow \frac{1-\varkappa}{\varkappa} = \frac{\ln\frac{T_2}{T_1}}{\ln\frac{p_1}{p_2}}$$

$$1 = \varkappa\left(1 + \frac{\ln\frac{T_2}{T_1}}{\ln\frac{p_1}{p_2}}\right)$$

$$\varkappa = \left(1 + \frac{\ln\frac{T_2}{T_1}}{\ln\frac{p_1}{p_2}}\right)^{-1} = 1{,}393$$

(37) $m = 1\,kg \qquad p_1 = 1$ bar $\qquad T_1 = 20\,°C = konst \qquad R = 287\,\frac{J}{kgK}$

$p_2 = 4$ bar

$$l_{12} = -\int_{v_1}^{v_2} p(v)\,dv = -\int_{v_1}^{v_2}\frac{RT}{v}\,dv = -RT\ln\frac{v_2}{v_1} \underset{p_1v_1 = p_2v_2}{=} RT\ln\frac{p_2}{p_1} = 116{,}575\,kJ$$

$$q_{12} = -l_{12}$$

$$u_2 - u_1 = c_v(T_2 - T_1) = 0 = l_{12} + q_{12}$$

(38) $m = 1\,kg$

$p_1 = 1\,bar$ $\quad T_1 = 20\,°C \quad R = 287\,\frac{J}{kg\,K} \quad \kappa = 1{,}4$

$p_2 = 4\,bar \quad c_v = \frac{R}{\kappa-1} = 717{,}5\,\frac{J}{kg\,K}$

$$V_1 = \frac{m R T_1}{p_1} = 0{,}84\ m^3 \qquad V_2 = V_1 \left(\frac{p_1}{p_2}\right)^{\frac{1}{\kappa}} = 0{,}3121\ m^3$$

$$T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{\frac{\kappa-1}{\kappa}} = 435{,}4\ K$$

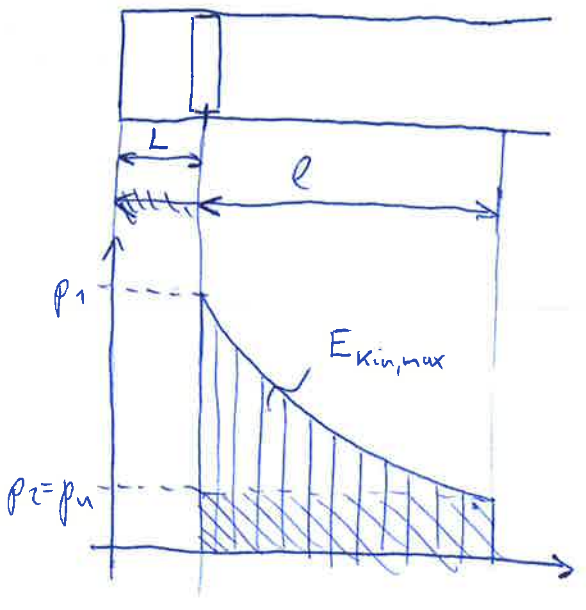
$$\ell_{12} = \frac{R}{\kappa-1}(T_2 - T_1) = 102{,}172\ kJ$$

$$q_{12} = 0 \qquad u_2 - u_1 = c_v (T_2 - T_1) = q_{12} + \ell_{12} = 102{,}172\ kJ$$

(39) $A = 20\,mm^2 \quad m = 2\,g \quad L = 40\,mm$

$p_1 = 80\,bar \quad T_1 = 300\,K \quad R = 286\,\frac{J}{kg\,K}$

$p_u = 1\,bar \quad T_u = 300\,K \quad \kappa = 1{,}4 \qquad p V^{\kappa} = konst$



$$L_{12} = -\int_{V_1}^{V_2} p(V)\, dV =$$

$$= -p_1 V_1^{\kappa}\, \frac{V_2^{\kappa-1} - V_1^{\kappa-1}}{\kappa - 1} = -39{,}9577\ J$$

$$-L_{12} = \underbrace{m\,\frac{w_k^2}{2}}_{\text{kinetische Energie des Kolbens}} + \underbrace{p_u (V_2 - V_1)}_{\text{Verdrängung der Luft aus dem Rohr}}$$

$$p_2 = p_u \Rightarrow V_2 = \underbrace{V_1}_{AL} \left(\frac{p_1}{p_2}\right)^{\frac{1}{\kappa}} = 18{,}2995 \cdot 10^{-6}\ m^3$$

$$\ell = \frac{V_2 - V_1}{A} = 0{,}875\ m$$

$$m\,\frac{w_k^2}{2} = -L_{12} - p_u (V_2 - V_1) = 38{,}2078\ J \Rightarrow w_{k,max} = 195{,}468\ \frac{m}{s}$$

(40) $m = 1\,kg$

$T_1 = 100\,°C$ $\quad T_2 = 300\,°C$ $\quad \kappa = 1{,}4$

$M_{O_2} = 32\ \frac{kg}{kmol}$ $\qquad R_{O_2} = \frac{R}{M_{O_2}} = 259{,}825\ \frac{J}{kg\,K}$

$c_v = \frac{R}{\kappa - 1} = 649{,}563\ \frac{J}{kg\,K}$

$c_p = \frac{\kappa R}{\kappa - 1} = 909{,}387\ \frac{J}{kg\,K}$

$Q_v = c_v m (T_2 - T_1) = 129{,}913\ J$

$Q_p = c_p m (T_2 - T_1) = 181{,}877\ J$

(1) $U_2 - U_1 = Q_v + \underbrace{L_{12,v}}_{\int_{V_1}^{V_2} p(V)\,dV = 0} = Q_v$ $\qquad U_2 - U_1 = c_v m (T_2 - T_1)$

(2) $U_2 - U_1 = Q_p + L_{12,p}$

(2) − (1):

$Q_p - Q_v = -L_{12,p} = \int_{V_1}^{V_2} p(V)\,dV = p(V_2 - V_1) \rightsquigarrow$ geleistete arbeit während der isobaren Zustandsänderung

(41) $V_1 = 10^{-3}\,m^3$ $\quad p_1 = 1\,bar$ $\quad T_1 = 20\,°C$

$p_2 = 12\,bar$

$\kappa = 1{,}4$ $\quad n = 1{,}3$ $\quad R = 287\ \frac{J}{kg\,K}$

$\nearrow c_v = \frac{R}{\kappa - 1} = 717{,}5\ \frac{J}{kg\,K}$

$\searrow c_p = \frac{\kappa R}{\kappa - 1} = 1004{,}5\ \frac{J}{kg\,K}$

$m = \frac{p_1 V_1}{R T_1} = 0{,}001189\ kg$

$T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = 519{,}888\ K$

$L_{12} = -\int_{V_1}^{V_2} p(V)\,dV = \ldots = m \frac{R}{n-1} (T_2 - T_1) = 258{,}08\ J$

$U_2 - U_1 = m c_v (T_2 - T_1) = 193{,}56\ J$

$H_2 - H_1 = m c_p (T_2 - T_1) = \kappa (U_2 - U_1) = 270{,}984\ J$

$$Q_{12} = (U_2 - U_1) - L_{12} = -64{,}52\,J = c_v \frac{n-\varkappa}{n-1}(T_2 - T_1)\,m$$

$$S_2 - S_1 = m\left[c_p \ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1}\right] = -0{,}163\,\frac{J}{K}$$

(42) $m = 1\,kg$

$Q_{12} = 20\,kJ$ $\qquad \varkappa = 1{,}4$ $\qquad V = konst$

$$\underbrace{U_2 - U_1}_{m c_v (T_2 - T_1)} = Q_{12} + \underbrace{L_{12}}_{-\int_{V_1}^{V_2} p(V)\,dV = 0} = Q_{12} \Rightarrow U_2 - U_1 = 20\,kJ$$

$$\underbrace{H_2 - H_1}_{m c_p (T_2 - T_1) = \varkappa(U_2 - U_1)} = Q_{12} + L_{12,t} \Rightarrow L_{12,t} = \underbrace{\varkappa(U_2 - U_1)}_{28\,kJ} - Q_{12} = 8\,kJ$$

(43) $m = 1\,kg$

$Q_{12} = 100\,kJ$ $\qquad \varkappa = 1{,}4$ $\qquad p = konst$

$$H_2 - H_1 = Q_{12} + \underbrace{L_{12,t}}_{\int_{p_1}^{p_2} V(p)\,dp = 0} = Q_{12} \Rightarrow H_2 - H_1 = 100\,kJ$$

$$U_2 - U_1 = \frac{1}{\varkappa}(H_2 - H_1) = Q_{12} + L_{12} \Rightarrow L_{12} = \underbrace{\frac{1}{\varkappa}(H_2 - H_1)}_{71{,}4286\,kJ} - Q_{12} = -28{,}5714\,kJ$$

(44) $d = 50\,mm$ $\qquad m_{CO_2} = 10\,g$ $\qquad T_1 = 300\,K$ $\qquad p_0 = 1\,bar$

$m_K = 5\,kg$ $\qquad Q_{zu} = 0{,}5\,kJ$ $\qquad g = 9{,}81\,\frac{m}{s^2}$

$R = 189\,\frac{J}{kg\,K}$ $\qquad c_v = 685\,\frac{J}{kg\,K}$

a) $$p_1 = p_0 + \frac{m_K g}{\frac{d^2 \pi}{4}} = 124{,}981 \ [kPa]$$

$$p = \text{konst} \Rightarrow \frac{T_2}{T_1} = \frac{v_2}{v_1} \Rightarrow v_2 = \frac{7}{6} v_1$$

$$w_{12} = -\int_{v_1}^{v_2} p \, dv = -p_1 (v_2 - v_1) = -\frac{1}{6} p_1 v_1 = -9450 \ \frac{J}{kg}$$

$$v_1 = \frac{R T_1}{p_1} = 0{,}4537 \left[\frac{m^3}{kg}\right]$$

$$W_{12} = m w_{12} = -94{,}5 \ J$$

$$U_2 - U_1 = Q_{12} + W_{12} \Rightarrow Q_{12} = (U_2 - U_1) - W_{12} =$$

$$= c_v m (T_2 - T_1) - W_{12} = 437 \ J$$

$$Q_{12} = Q_{zu} + Q_{ab} \Rightarrow Q_{ab} = 63 \ J$$

b) $V = \text{konst}$

$$U_2 - U_1 = Q_{12} + \underbrace{W_{12}}_{= -\int_{V_1}^{V_2} p \, dV = 0} \Rightarrow Q_{12} = U_2 - U_1 = c_v m (T_2 - T_1)$$

$$\Rightarrow T_2 = T_1 + \frac{Q_{12}}{c_v m} =$$

$$= 363{,}796 \ K$$

$$v = \text{konst} \Rightarrow \frac{T_2}{T_1} = \frac{p_2}{p_1} \Rightarrow p_2 = \frac{T_2}{T_1} p_1 = 151{,}558 \ kPa$$

(45) $p_1 = 1 bar$ $\quad p_2 = 100 bar$ $\quad \rho = 1000 \frac{kg}{m^3}$ $\quad m = 1000 kg$

inkompressibel: $v = konst$

$$L_{12} = -\int_{V_1}^{V_2} p(V)\,dV = 0$$

$$L_{12,t} = \int_{p_1}^{p_2} V(p)\,dp = \int_{p_1}^{p_2} \frac{m}{\rho}\,dp = \frac{m}{\rho}(p_2 - p_1) = 9{,}9\ MJ$$

(46) $v = A \cdot B$ $\quad A = a_0 + a_1 p + a_2 p^2$ $\quad m = 1000 kg$

$B = b_0 + b_1 T + b_2 T^2$

$$L_{12,t} = m\int_{p_1}^{p_2} v(p)\,dp = mB\int_{p_1}^{p_2} A\,dp =$$

$$= m\left[b_0 + b_1 T + b_2 T^2\right]\left[a_0(p_2 - p_1) + \frac{a_1}{2}(p_2^2 - p_1^2) + \frac{a_2}{3}(p_2^3 - p_1^3)\right] =$$

$$= 9{,}9987\ MJ$$

$$L_{12} = L_{12,t} - m(p_2 \underbrace{v_2}_{0{,}001007 \frac{m^3}{kg}} - p_1 \underbrace{v_1}_{0{,}001012 \frac{m^3}{kg}}) = 22190{,}5\ J \cong 22\ kJ$$

(47) $p_1 = 1 bar$ $\quad T_1 = 27°C$ $\quad \dot{V}_1 = 200 \frac{m^3}{h}$

$p_2 = 7 bar$ $\quad n = 1{,}3$ $\quad \kappa = 1{,}4$ $\quad R = 287 \frac{J}{kg\,K}$

$$\dot{m} = \frac{p_1 \dot{V}_1}{T_1 R} = 232{,}288 \frac{kg}{h} = 0{,}0645 \frac{kg}{s}$$

$$\dot{L}_{12,t} = \int_{p_1}^{p_2} \dot{V}(p)\,dp = \dot{m}\int_{p_1}^{p_2} v(p)\,dp = \overbrace{\dot{m} v_1}^{\dot{V}_1} p_1^{\frac{1}{n}} \frac{p_2^{1-\frac{1}{n}} - p_1^{1-\frac{1}{n}}}{1 - \frac{1}{n}} = 13{,}646\ kW$$

$$p_1 v_1^n = p v^n \Rightarrow v = v_1 \left(\frac{p_1}{p}\right)^{\frac{1}{n}}$$

$$\left.\begin{aligned} (1)\quad & p v^n = konst \\ (2)\quad & \frac{p v}{T} = konst \end{aligned}\right\} \quad \frac{(1)}{(2)^n}: \ T^n p^{1-n} = konst$$

$$\frac{T}{p^{\frac{n-1}{n}}} = konst \Rightarrow T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = 470{,}05\ K$$

(48) $V_H = 1\,dm^3$ $\quad p_0 = 1\,bar$ $\quad T_0 = 300\,K$

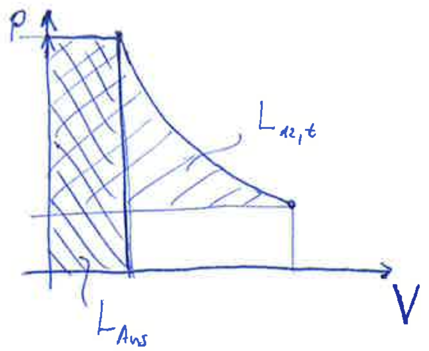
$p_1 = 12\,bar$ $\quad R = 287\,\frac{J}{kg\,K}$ $\quad n = 1{,}25$

a) $m = \frac{p_0 V_H}{T_0 R} = 0{,}0011161\,kg$

b) $T_1 = T_0 \left(\frac{p_1}{p_0}\right)^{\frac{n-1}{n}} = 493{,}126\,K$

c) $L_{12,t} = \int_{p_1}^{p_2} V(p)\,dp = V_H\, p_0^{\frac{1}{n}} \frac{p_1^{\frac{n-1}{n}} - p_0^{\frac{n-1}{n}}}{\frac{n-1}{n}} = 321{,}876\,J/\text{Umdrehung}$

d)



(49) $\dot m = 0{,}1\,\frac{kg}{s}$ $\quad p_1 = 1\,bar$ $\quad T_1 = 300\,K$ $\quad p_2 = 5\,bar$

$n = 1{,}25$ $\quad c_p = 1\,\frac{kJ}{kg\,K}$ $\quad R = 287\,\frac{J}{kg\,K}$

$T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = 413{,}919\,K$

$\dot L_{12,t} = \int_{p_1}^{p_2} \dot V(p)\,dp = \dot m \int_{p_1}^{p_2} v(p)\,dp = \dot m\, v_1\, p_1^{\frac{1}{n}} \frac{p_2^{\frac{n-1}{n}} - p_1^{\frac{n-1}{n}}}{\frac{n-1}{n}} = 16{,}3474\,kW$

$v_1 = \frac{R T_1}{p_1} = 0{,}861\,\frac{m^3}{kg}$

$\dot H_2 - \dot H_1 = c_p\, \dot m\, (T_2 - T_1) = 11{,}399\,kW$

$\dot Q_{12} = \dot H_2 - \dot H_1 - \dot L_{12,t} = -4{,}9484\,kW$

(50) $m = 1\,kg$

$p_1 = 100\,bar$ $\quad v_1 = 0{,}036\,\frac{m^3}{kg}$ $\quad h_1 = 3490\,\frac{kJ}{kg}$

$p_2 = 15\,bar$ $\quad v_2 = 0{,}16\,\frac{m^3}{kg}$ $\quad h_2 = 2930\,\frac{kJ}{kg}$

$$h_2 - h_1 = q_{12} + \ell_{12,t} \Rightarrow \ell_{12,t} = h_2 - h_1 = -560\,\frac{kJ}{kg}$$

$$\ell_{12} = \ell_{12,t} - p_2 v_2 + p_1 v_1 = -440\,\frac{kJ}{kg}$$

(51) $\dot{m} = 500\,\frac{kg}{h}$ $\quad p_1 = 2\,bar$ $\quad T_1 = 700\,°C$

$p_2 = 1\,bar$ $\quad \kappa = 1{,}4$ $\quad c_p = 1\,\frac{kJ}{kg\,K}$

$$R = c_p - c_v = c_p\left(1 - \frac{1}{\kappa}\right) = 285{,}714\,\frac{J}{kg\,K}$$

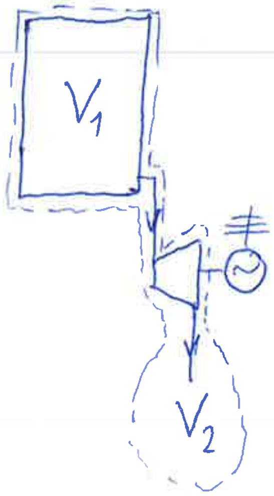
$$v_1 = \frac{R T_1}{p_1} = 1{,}39\,\frac{m^3}{kg}$$

$$\dot{L}_{12,t} = \dot{m}\, v_1\, p_1^{\frac{1}{\kappa}}\,\frac{p_2^{\frac{\kappa-1}{\kappa}} - p_1^{\frac{\kappa-1}{\kappa}}}{\frac{\kappa-1}{\kappa}} = -24{,}2797\,kW$$

(52) $V = 100\,\ell$ $\quad p_1 = 10\,bar$ $\quad T_1 = 80\,°C$

$p_0 = 1{,}013\,bar$

$c_p = 1\,\frac{kJ}{kg\,K}$ $\quad \kappa = 1{,}4$

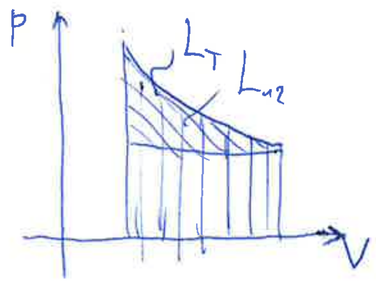


1. Lösung: geschlossenes System

$$V_2 = V_1\left(\frac{p_1}{p_0}\right)^{\frac{1}{\kappa}} = 0{,}5132\,m^3$$

$$T_2 = T_1\left(\frac{p_0}{p_1}\right)^{\frac{\kappa-1}{\kappa}} = 183{,}511\,K$$

$$L_{12} = -\int_{V_1}^{V_2} p(V)\,dV = -p_1 V_1^{\kappa}\int_{V_1}^{V_2} V^{-\kappa}\,dV = -p_1 V_1^{\kappa}\,\frac{V_2^{1-\kappa} - V_1^{1-\kappa}}{1-\kappa} = -120{,}055\,kJ$$



$$|L_T| = |L_{12}| - p_0\,(V_2 - V_1) = 78{,}1778\,kJ$$

2. Lösung: offenes System

$$R = c_p\left(1-\frac{1}{\varkappa}\right) = 285{,}714\ \frac{J}{kg\,K}$$

$$v_1 = \frac{RT_1}{p_1} = 0{,}101\ \frac{m^3}{kg}$$

$$p v^{\varkappa} = konst \Rightarrow v = v_1\left(\frac{p_1}{p}\right)^{\frac{1}{\varkappa}}$$

$$\frac{T}{p^{\frac{\varkappa-1}{\varkappa}}} = konst \Rightarrow T = T_1\left(\frac{p}{p_1}\right)^{\frac{\varkappa-1}{\varkappa}} \qquad T_2 = T_1\left(\frac{p_0}{p_1}\right)^{\frac{\varkappa-1}{\varkappa}}$$

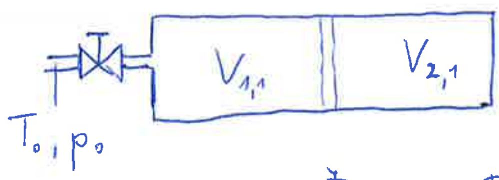
$$m(p) = \frac{V}{v} = \frac{V}{v_1\left(\frac{p_1}{p}\right)^{\frac{1}{\varkappa}}} = \frac{V}{v_1\, p_1^{1/\varkappa}}\, p^{1/\varkappa}$$

$$dm = \frac{V}{\varkappa v_1\, p_1^{1/\varkappa}}\, p^{\frac{1}{\varkappa}-1}\, dp$$

$$(h_2 - h_0)\,dm = dL_T$$

$$\int_m c_p(T_2 - T_0)\,dm = \int_{p_1}^{p_0} c_p \frac{T_1}{p_1^{\frac{\varkappa-1}{\varkappa}}}\left[p_0^{\frac{\varkappa-1}{\varkappa}} - p^{\frac{\varkappa-1}{\varkappa}}\right] \frac{V}{\varkappa v_1\, p_1^{\frac{1}{\varkappa}}}\, p^{\frac{1-\varkappa}{\varkappa}}\, dp = -78{,}067{,}6\ kJ$$

53



$V_{1,1} = 2\,l$ $\qquad T_{1,1} = 300\,K$ $\qquad p_{1,1} = 1\,bar$

$V_{2,1} = 2\,l$ $\qquad T_{2,1} = 373\,K \rightsquigarrow p_{2,1} = 1\,bar$

$T_0 = 300\,K$ $\qquad p_0 = 7\,bar$

$c_{p,L} = 1001\ \frac{J}{kg\,K}$ $\qquad R_L = 287\ \frac{J}{kg\,K}$ $\qquad c_{p,D} = 1900\ \frac{J}{kg\,K}$ $\qquad R_D = 460\ \frac{J}{kg\,K}$

a) $$m_D = \frac{p_{2,1}\, V_{2,1}}{R\, T_{2,1}} = 0{,}001166\ kg = 1{,}166\ g$$

b) $$T_{2,2} = T_{2,1}\left(\frac{p_0}{p_{2,1}}\right)^{\frac{\varkappa-1}{\varkappa}} = 597{,}434\,K$$

$$R_D = c_{p,D} - c_{v,D}$$

$$\varkappa_D = \frac{c_{p,D}}{c_{v,D}} = \frac{c_{p,D}}{c_{p,D} - R_D} = 1{,}3194$$

c) $V_{2,2} = V_{2,1} \left(\frac{p_{2,1}}{p_0}\right)^{\frac{1}{\varkappa}} = 0{,}4576 \ \ell$

$$L_{12,D} = m_D \int_{v_{2,1}}^{v_{2,2}} p(v)\, dv = m_D \int_{v_{2,1}}^{v_{2,2}} \frac{v_{2,1}^{\varkappa}\, p_{2,1}}{v^{\varkappa}}\, dv = \ldots =$$

$$= m_D \frac{R}{\varkappa - 1} \left(T_{2,2} - T_{2,1}\right) = 376{,}886 \ J$$

d) $c_{v,L} = c_{p,L} - R_L = 714 \ \frac{J}{kg\,K}$

$$m_{L,1} = \frac{p_{1,1} V_{1,1}}{R_L T_{1,1}} = 0{,}002323 \ kg$$

$$V_{1,2} = V_{1,1} + \left(V_{2,1} - V_{2,2}\right) = 3{,}5424 \ \ell$$

instationäres, offenes System:

$$U_{1,2} - U_{1,1} = H_{ein} - L_{12,D}$$

$$\left(m_{L,1} + m_{ein}\right) c_{v,L} T_{1,2} - m_{L,1} c_{v,L} T_{1,1} = m_{ein}\, c_{p,L}\, T_0 - L_{12,D}$$

$$\left.\begin{aligned} m_{L,1} c_{v,L}\left(T_{1,2} - T_{1,1}\right) + m_{ein}\, c_{v,L}\, T_{1,2} &= m_{ein}\, c_{p,L}\, T_0 - L_{12,D} \\ m_{L,2} = m_{L,1} + m_{ein} &= \frac{p_{1,2} V_{1,2}}{R\, T_{1,2}} \end{aligned}\right\} \text{Gleichungssystem für } T_{1,2},\ m_{ein}$$

$$\frac{p_{1,2} V_{1,2}}{R\, \cancel{T_{1,2}}} c_{v,L}\, \cancel{T_{1,2}} - m_{L,1} c_{v,L} T_{1,1} = \left(\frac{p_{1,2} V_{1,2}}{R\, T_{1,2}} - m_{L,1}\right) c_{p,L}\, T_0 - L_{12,D}$$

$$\frac{p_{1,2} V_{1,2}}{R} c_{v,L} - m_{L,1} c_{v,L} T_{1,1} + m_{L,1} c_{p,L} T_0 + L_{12,D} = \frac{p_{1,2} V_{1,2}}{R\, T_{1,2}} c_{p,L}\, T_0$$

$$T_{1,2} = \frac{p_{1,2} V_{1,2} T_0 c_{p,L}}{R} \left[\frac{p_{1,2} V_{1,2}}{R} c_{v,L} - m_{L,1} c_{v,L} T_{1,1} + m_{L,1} c_{p,L} T_0 + L_{12,D}\right]^{-1} =$$

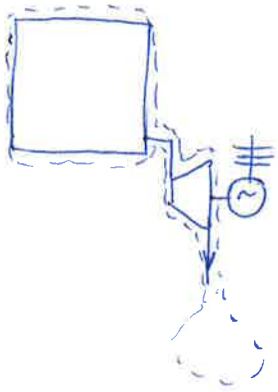
$$= 384{,}616 \ K$$

$$m_{ein} = \frac{p_{1,2} V_{1,2}}{R\, T_{1,2}} - m_{L,1} = 0{,}020138 \ kg$$

$$m_{L,2} = 0{,}022461 \ kg$$

(54) $T_1 = 20\,°C$ $\quad p_1 = 2\,bar$ $\quad p_u = 1\,bar$

$m_{aus} = 20\,kg$ $\quad R = 287\,\frac{J}{kg\,K}$

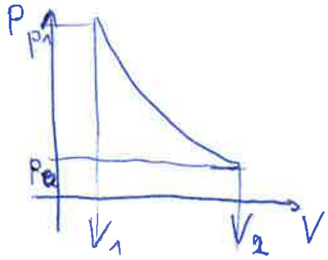


isotherm: $pV = konst \Rightarrow p_1 V_1 = p_u V_2$

(1) $p_1 V_1 = m_1 R T$ $\quad \cancel{\Rightarrow V_1 = \frac{m_1 RT}{p_1} = 8{,}409\,m^3}$

(2) $p_u V_1 = (m_1 - m_{aus}) RT$

$$\frac{(1)}{(2)}: \frac{p_1}{p_u} = \frac{m_1}{m_1 - m_{aus}} \Rightarrow m_1 = \frac{\frac{p_1}{p_u} m_{aus}}{\frac{p_1}{p_u} - 1} = 40\,kg$$



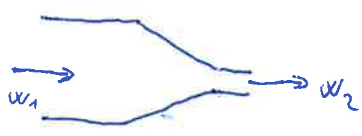
$$V_1 = \frac{m_1 RT}{p_1} = 16{,}8182\,m^3 \leadsto V_2 = \frac{p_1}{p_u} V_1 = 33{,}6364$$

$$L_{12} = -\int_{V_1}^{V_2} p(V)\,dV = -\int_{V_1}^{V_2} \frac{mRT}{V}\,dV = -m_{aus} RT \ln\frac{V_2}{V_1} = -m_{aus} RT \ln\frac{p_1}{p_0} =$$

$$= -1{,}1658\,MJ = -Q_{12} \quad (1.\,HS)$$

(55) $w_1 = 10\,\frac{m}{s}$ $\quad c_p = 1\,\frac{kJ}{kg\,K}$

$w_2 = 80\,\frac{m}{s}$



wärmeisolierte Düse $\rightarrow q_{12} = 0$

- $(h_2 - h_1) + \frac{1}{2}(w_2^2 - w_1^2) + \underbrace{g(z_2 - z_1)}_{=0} = \underbrace{q_{12}}_{=0} + \underbrace{l_{12,t}}_{=0}$
- $h_2 - h_1 = c_p (T_2 - T_1) = -\frac{1}{2}(w_2^2 - w_1^2) \Rightarrow T_2 - T_1 = -\frac{1}{2c_p}(w_2^2 - w_1^2) = -4\,K$
- $\underbrace{(h_2 - h_1)}_{\underbrace{c_p(T_2 - T_1)}_{=0}} + \frac{1}{2}(w_2^2 - w_1^2) + \underbrace{g(z_2 - z_1)}_{=0} = q_{12} + \underbrace{l_{12,t}}_{=0}$

$$q_{12} = \frac{1}{2}(w_2^2 - w_1^2) = 4\,kJ$$

(56) $p_1 = 2\,bar$ $\qquad T_1 = 20\,°C$ $\qquad w_1 = 150\,\frac{m}{s}$ $\qquad w_2 = 10\,\frac{m}{s}$

$c_p = 1\,\frac{kJ}{kg K}$ $\qquad c_v = 0{,}714\,\frac{kJ}{kg K}$

$$(h_2 - h_1) + \frac{1}{2}(w_2^2 - w_1^2) + \underbrace{g(z_2 - z_1)}_{=0} = \underbrace{q_{12}}_{=0} + \underbrace{l_{12,t}}_{=0}$$

$$T_2 = T_1 - \frac{1}{2c_p}(w_2^2 - w_1^2) = 31{,}2\,°C$$

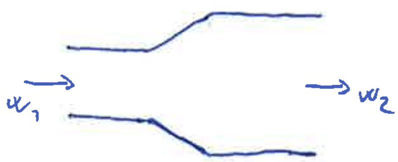
reversible: $s_2 - s_1 = 0 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1}$

$$\ln\left(\frac{T_2}{T_1}\right)^{c_p} = \ln\left(\frac{p_2}{p_1}\right)^{R}$$

$$p_2 = p_1 \left(\frac{T_2}{T_1}\right)^{\frac{c_p}{R}} = p_1 \left(\frac{T_2}{T_1}\right)^{\frac{\kappa}{\kappa-1}} = 2{,}28061\,bar$$

(57) $\dot{m} = 200\,\frac{kg}{s}$ $\qquad A_1 = 0{,}01\,m^2$ $\qquad A_2 = 0{,}04\,m^2$

$\rho = 1000\,\frac{kg}{m^3}$ $\qquad c_W = c_p = c_v = 4{,}2\,\frac{kJ}{kg K}$



a) Kontinuitätsgleichung: $\dot{m} = w_1 A_1 \rho_1 = w_2 A_2 \rho_2 \Rightarrow w_1 = \frac{\dot{m}}{A_1 \rho} = 20\,\frac{m}{s}$

$$w_2 = \frac{\dot{m}}{A_2 \rho} = 5\,\frac{m}{s}$$

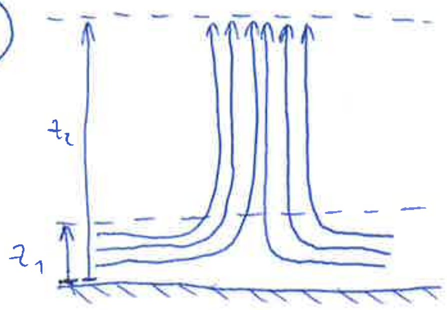
1. HS: $(h_2 - h_1) + \frac{1}{2}(w_2^2 - w_1^2) + \underbrace{g(z_2 - z_1)}_{=0} = \underbrace{q_{12}}_{=0} + \underbrace{l_{12,t}}_{=0}$

$$T_2 - T_1 = -\frac{1}{2c_p}(w_2^2 - w_1^2) = 0{,}045\,K$$

$$l_{12,t} = \underbrace{\int_{p_1}^{p_2} v(p)\,dp}_{\frac{1}{\rho}(p_2 - p_1)} + \underbrace{|l_R|}_{=0} + \frac{1}{2}(w_2^2 - w_1^2) + \underbrace{g(z_2 - z_1)}_{=0} = 0$$

$$p_2 - p_1 = -\frac{\rho}{2}(w_2^2 - w_1^2) = 187\,500\,Pa$$

(57a)



$z_1 = 200\,m$ $\quad T_1 = 300\,K$ $\quad p_1 = 1\,bar$

$z_2 = 700\,m$

$c_p = 1000\,\frac{J}{kg\,K}$ $\quad \kappa = 1{,}4$

$w_1 = w_2$

$$\underbrace{(h_2 - h_1)}_{c_p(T_2-T_1)} + \frac{1}{2}\underbrace{(w_2^2 - w_1^2)}_{=0} + g(z_2 - z_1) = \underbrace{q_{12}}_{=0} + \underbrace{l_{12,t}}_{=0}$$

$$T_2 = T_1 - \frac{g}{c_p}(z_2 - z_1) = 295{,}095\,K$$

$$s_2 - s_1 = 0 = c_p \ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1} \Rightarrow p_2 = p_1\left(\frac{T_2}{T_1}\right)^{\frac{\kappa}{\kappa-1}} = 0{,}9439\,bar$$

(58) $T_1 = 0\,°C$ $\quad T_2 = 900\,°C$ $\quad w_1 = 40\,\frac{m}{s}$ $\quad p_1 = 10\,bar$

$M = 28\,\frac{kg}{kmol}$ $\quad \kappa = 1{,}4$

$$R = \frac{\mathcal{R}}{M} = 296{,}943\,\frac{J}{kg\,K} \qquad c_p = \frac{\kappa R}{\kappa - 1} = 1039{,}3\,\frac{J}{kg\,K}$$

$$\left.\begin{array}{ll}
(1) & c_p(T_2 - T_1) + \frac{1}{2}(w_2^2 - w_1^2) + g\underbrace{(z_2 - z_1)}_{=0} = q_{12} + \underbrace{l_{12,t}}_{=0} \\
(2) & \frac{w_1}{v_1}(w_2 - w_1) = p_1 - p_2 \\
(3) & \frac{w_1}{v_1} = \frac{w_2}{v_2} \\
(4) & v_1 = \frac{RT_1}{p_1} \\
(5) & v_2 = \frac{RT_2}{p_2}
\end{array}\right\}$$

5 Gleichungen für die $v_1, v_2, w_2, p_2, q_{12}$ Unbekannten

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(4) $v_1 = \frac{RT_1}{p_1} = 0{,}081065\,\frac{m^3}{kg}$

(3) $v_2 = v_1\frac{w_2}{w_1}$

(5) $v_1\frac{w_2}{w_1} = \frac{RT_2}{p_2} \Rightarrow w_2 = \frac{RT_2}{v_1 p_2}w_1$

(2) $$\frac{w_1}{v_1}\left(\frac{RT_2}{v_1 p_2} w_1 - w_1\right) = p_1 - p_2$$

$$p_2^2 - \left(\frac{w_1^2}{v_1} + p_1\right) p_2 + \frac{w_1^2 RT_2}{v_1^2} = 0 \quad \Rightarrow \quad p_2 = \begin{cases} 91345{,}8 \ Pa \\ \boxed{928\ 391 \ Pa} \end{cases}$$

(5) $$v_2 = \frac{RT_2}{p_2} = \begin{cases} 3{,}81313 \ \frac{m^3}{kg} \\ \boxed{0{,}37518 \ \frac{m^3}{kg}} \end{cases}$$

(3) $$w_2 = \frac{v_2}{v_1} w_1 = \begin{cases} 1881{,}51 \ \frac{m}{s} \\ \boxed{185{,}125 \ \frac{m}{s}} \end{cases}$$

(1) $$q_{12} = c_p (T_2 - T_1) + \frac{1}{2}(w_2^2 - w_1^2) = \begin{cases} 2{,}7046 \ \frac{MJ}{kg} \\ \boxed{951{,}706 \ \frac{kJ}{kg}} \end{cases}$$

falls $p_2 = p_1$

$$v_1 = \frac{RT_1}{p} = 0{,}081065 \ \frac{m^3}{kg} \qquad v_2 = \frac{RT_2}{p_2} = 0{,}348314 \ \frac{m^3}{kg}$$

$$w_2 = \frac{v_2}{v_1} w_1 = \frac{T_2}{T_1} w_1 = 171{,}869 \ \frac{m}{s}$$

$$q_{12} = c_p (T_2 - T_1) + \frac{1}{2}(w_2^2 - w_1^2) = 949{,}339 \ kJ$$

(59) $w_1 = 50 \ \frac{m}{s}$ $\quad p_1 = 10 \ bar$ $\quad T_1 = 27\ °C$ $\quad \rho_1 = 10 \ \frac{kg}{m^3}$

$c_p = 1 \ \frac{kJ}{kg K}$ $\quad q_{12} = 0$ $\quad p_2 = 2 \ bar$ $\quad |l_R| \neq 0$

$$R = \frac{p_1}{\rho_1 T_1} = 333{,}33 \ \frac{J}{kg K}$$

$$(h_2 - h_1) + \frac{1}{2}(w_2^2 - w_1^2) + g\underbrace{(z_2 - z_1)}_{=0} = \underbrace{q_{12}}_{=0} + \underbrace{l_{n,t}}_{=0}$$

$$\dot m_1 = \dot m_2 = \rho_1 A_1 w_1 = \rho_2 A_2 w_2$$

$$R = \frac{p_1}{\rho_1 T_1} = \frac{p_2}{\rho_2 T_2} \quad , \quad h_2 - h_1 = c_p (T_2 - T_1)$$

$$\Rightarrow \begin{cases} c_p (T_2 - T_1) = -\frac{1}{2} (w_2^2 - w_1^2) \\ w_1 \rho_1 = w_2 \rho_2 \\ \frac{p_1}{\rho_1 T_1} = \frac{p_2}{\rho_2 T_2} \end{cases}$$

für die T_2, ρ_2, w_2 Variablen

$$w_2 = w_1 \frac{\rho_1}{\rho_2} = w_1 \frac{p_1 T_2}{p_2 T_1} \Rightarrow c_p (T_2 - T_1) = -\frac{1}{2} w_1^2 \left[\left(\frac{p_1 T_2}{p_2 T_1} \right)^2 - 1 \right]$$

$$\frac{\rho_1}{\rho_2} = \frac{p_1 T_2}{p_2 T_1}$$

$$\Rightarrow \left(\frac{1}{2 c_p} w_1^2 \frac{p_1^2}{p_2^2 T_1^2} \right) T_2^2 + T_2 - \left(T_1 + \frac{1}{2 c_p} w_1^2 \right) = 0$$

$$T_2 = \begin{cases} -3154{,}99 \text{ K} \\ 274{,}993 \text{ K} \end{cases} \Rightarrow \rho_2 = \rho_1 \frac{p_2 T_1}{p_1 T_2} = 2{,}18187 \ \frac{kg}{m^3}$$

$$\Rightarrow w_2 = w_1 \frac{\rho_1}{\rho_2} = 229{,}161 \ \frac{m}{s}$$

$$s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1} = 449{,}437 \ \frac{J}{kg\,K}$$

(60) $p_1 = 10$ bar $\quad T_1 = 5\,°C \quad w_1 = 10 \ \frac{m}{s} \quad c_p = 1004 \ \frac{J}{kg\,K}$

$p_2 = 1$ bar $\quad q_{12} = 0 \quad c_v = 715 \ \frac{J}{kg\,K}$

$$R = c_p - c_v = 289 \ \frac{J}{kg\,K} \qquad \varkappa = \frac{c_p}{c_v} = 1{,}4$$

$$v_1 = \frac{R T_1}{p_1} = 0{,}080342 \ \frac{m^3}{kg}$$

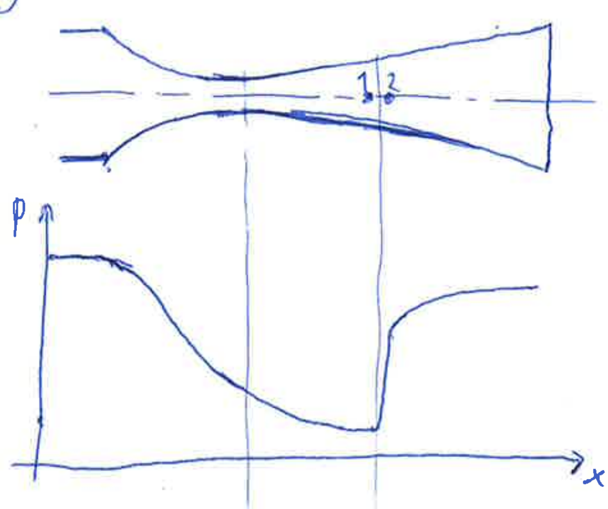
$$\begin{cases} c_p\,(T_2 - T_1) = -\frac{1}{2}\left(w_2^2 - w_1^2\right) \\[4pt] \dfrac{w_1}{v_1} = \dfrac{w_2}{v_2} \\[4pt] \dfrac{p_1 v_1}{T_1} = \dfrac{p_2 v_2}{T_2} \end{cases} \Rightarrow T_2 = \begin{cases} -15791{,}9\,K \quad \swarrow N \\ 273{,}239\,K \end{cases}$$

$$v_2 = 0{,}8174419\ \frac{m^3}{kg}$$

$$w_2 = 101{,}742\ \frac{m}{s}$$

$$s_2 - s_1 = c_p \ln\frac{T_2}{T_1} - R \ln\frac{p_2}{p_1} = 648{,}104\ \frac{J}{kg\,K}$$

(61) z.B. Lavaldüse



$$p_1 = 1\,bar$$

$$T_1 = 27^\circ C$$

$$\rho_1 = 1\ \frac{kg}{m^3}$$

$$c_p = 1\ \frac{kJ}{kg\,K}$$

$$R = \frac{p_1}{\rho_1 T_1} = 333{,}33\ \frac{J}{kg\,K}$$

$$dh = c_p\,dT$$

$$\begin{cases} (h_2 - h_1) + \frac{1}{2}\left(w_2^2 - w_1^2\right) + g\underbrace{(z_2 - z_1)}_{=0} = \underbrace{q_{12}}_{=0} + \underbrace{l_{12,t}}_{=0} & (1) \\[4pt] \dot m_1 = \dot m_2 = \rho_1 w_1 A_1 = \rho_2 w_2 A_2 \qquad A_1 \approx A_2 & (2) \\[4pt] \dot m\,(w_2 - w_1) = p_1 A_1 - p_2 A_2 & (3) \\[4pt] \dfrac{p_1}{\rho_1 T_1} = \dfrac{p_2}{\rho_2 T_2} & (4) \end{cases}$$

$$(1):\quad T_2 = T_1 - \frac{1}{2c_p}\left(w_2^2 - w_1^2\right) \underset{\rho_1 w_1 = \rho_2 w_2}{=} T_1 - \frac{1}{2c_p} w_1^2 \left[\left(\frac{\rho_1}{\rho_2}\right)^2 - 1\right]$$

$$(3):\quad \rho_1 w_1^2 \left(\frac{\rho_1}{\rho_2} - 1\right) = p_1\left(1 - \frac{p_2}{p_1}\right) \underset{\frac{p_2}{p_1} = \frac{\rho_2 T_2}{\rho_1 T_1}}{=} p_1\left(1 - \frac{\rho_2 T_2}{\rho_1 T_1}\right) \Rightarrow \left[1 - \frac{\rho_1 w_1^2}{p_1}\left(\frac{\rho_1}{\rho_2} - 1\right)\right]\frac{\rho_1}{\rho_2} = \frac{T_2}{T_1}$$

$$(1) \quad \frac{T_2}{T_1} = 1 - \frac{1}{2c_pT_1} w_1^2 \left[\left(\frac{\rho_1}{\rho_2}\right)^2 - 1\right] \underset{(3)}{=} \left[1 - \frac{\rho_1 w_1^2}{p_1}\left(\frac{\rho_1}{\rho_2} - 1\right)\right]\frac{\rho_1}{\rho_2}$$

$$\left(\frac{\rho_1 w_1^2}{p_1} - \frac{1}{2c_pT_1} w_1^2\right)\left(\frac{\rho_1}{\rho_2}\right)^2 - \left(1 + \frac{\rho_1 w_1^2}{p_1}\right)\frac{\rho_1}{\rho_2} + \left(1 + \frac{1}{2c_pT_1} w_1^2\right) = 0$$

$$\frac{\rho_1}{\rho_2} = \begin{cases} 0{,}68 & \Rightarrow \quad \rho_2 = 1{,}47 \ \frac{kg}{m^3} \\ 1 & \Rightarrow \quad \rho_2 = 1 \ \frac{kg}{m^3} \end{cases}$$

$$(3) \quad \frac{T_2}{T_1} = \begin{cases} 1{,}224 & \Rightarrow \quad T_2 = 367{,}2 \ K \\ 1 & \Rightarrow \quad T_2 = 300 \ K \end{cases}$$

$$p_2 = p_1 \frac{\rho_2 T_2}{\rho_1 T_1} = \begin{cases} 1 \ bar \\ 1{,}8 \ bar \end{cases}$$

$$w_2 = w_1 \frac{\rho_1}{\rho_2} = \begin{cases} 500 \ \frac{m}{s} \\ 340 \ \frac{m}{s} \end{cases}$$

$$s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1} = 6{,}19726 \ \frac{J}{kg\,K}$$

(62) $p = 100$ bar $\qquad V = 1\ dm^3 \qquad L_V = -pV = -100 \cdot 10^5 [bar] \cdot 10^{-3} [m^3] = -10\ kJ$

(63) $p = 100$ bar $\qquad \dot V = 1 \frac{dm^3}{s}$

a) $\dot H_2 - \dot H_1 = \underbrace{\dot Q_{12}}_{=0} + \underbrace{\dot L_{12,t}}_{=0} \equiv 0 \Rightarrow \dot U_2 - \dot U_1 = -p(\dot V_2 - \dot V_1) = 0 \longrightarrow$ nein

b) $\dot L_V = 10 \ \frac{kJ}{s} = 10\ kW$

c) $\dot H_2 - \dot H_1 = \underbrace{\dot Q_{12}}_{=0} + \underbrace{\dot L_{12,t}}_{=0} \equiv 0 \Rightarrow \dot L_{12,t} = \dot L_V + |\dot L_R|$

(64) $T = \text{konst}$ $\quad u = u(T, v)$ $\quad p = p(T, v)$

$$du = \frac{\partial u}{\partial T}\Big|_v \underbrace{dT}_{=0} + \frac{\partial u}{\partial v}\Big|_T dv = \frac{\partial u}{\partial v}\Big|_T dv \Rightarrow u_2 - u_1 = \int_{v_1}^{v_2} \frac{\partial u}{\partial v}\Big|_T dv$$

$$d\ell = -p\,dv \Rightarrow \ell_{12} = -\int_{v_1}^{v_2} p(T=\text{konst}, v)\,dv$$

$$q_{12} = (u_2 - u_1) - \ell_{12} = \int_{v_1}^{v_2} \left(\frac{\partial u}{\partial v}\Big|_T + p \right) dv$$

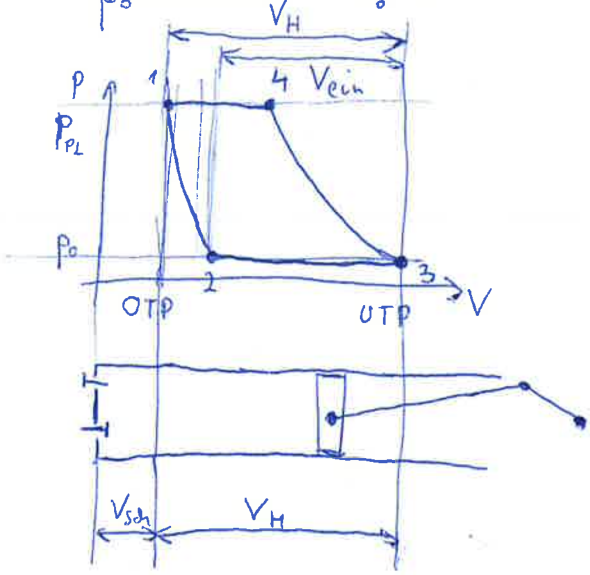
(65)
$$U_2 - U_1 = m \int_{v_1}^{v_2} \frac{\partial u}{\partial v}\Big|_T dv \underset{\text{Trapezregel}}{\cong} \frac{m}{2} \sum_{k=1}^{n-1} (v_{k+1} - v_k)\left[\left(\frac{\partial u}{\partial v}\Big|_T\right)_{k+1} + \left(\frac{\partial u}{\partial v}\Big|_T\right)_k\right] = 293{,}825 \text{ kJ}$$

$$L_{12} = -m \int_{v_1}^{v_2} p\,dv \cong -\frac{m}{2} \sum_{\ell=1}^{n-1} (v_{\ell+1} - v_\ell)(p_{\ell+1} + p_\ell) = -372{,}5 \text{ kJ}$$

$$Q_{12} = (U_2 - U_1) - L_{12} = 666{,}325 \text{ kJ}$$

(66) $d = 120$ mm $\quad \ell = 150$ mm $\quad V_{sch} = 170\text{ cm}^3$ $\quad T_1 = 137{,}6\ ^\circ C$

$p_0 = 1$ bar $\quad T_0 = 27\ ^\circ C$ $\quad p_{PL} = 3$ bar $\quad c_p = 1 \frac{kJ}{kg\,K}$ $\quad \varkappa = 1{,}4$



$$V_H = \frac{d^2\pi}{4}\ell = 0{,}00169646 \text{ m}^3$$

$$V = V_H + V_{sch} = 0{,}00186646 \text{ m}^3$$

1-2: Einsaugventil geschlossen
2-3: Einsaugen, Einsaugventil geöffnet
3-4: Komprimieren
4-1: Ausströmung die komprimierte Luft

$$T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{\frac{\varkappa-1}{\varkappa}} = 299{,}984 \text{ K} \cong 300 \text{ K} = T_0$$

$$V_2 = V_1 \left(\frac{p_1}{p_2}\right)^{\frac{1}{\varkappa}} = 0{,}000372606 \text{ m}^3 = V_{zyl} \text{ in Zylinder}$$

$$V_{ein} = V - V_2 = 0{,}00149385 \text{ m}^3$$

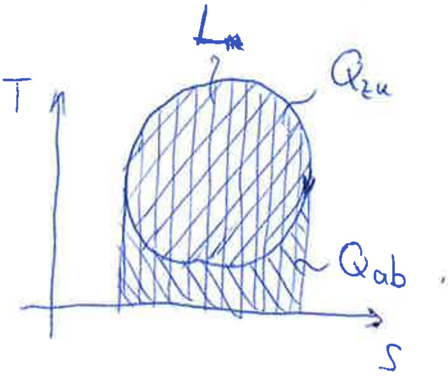
$$\eta_v = \frac{V_{ein}}{V_H} = 0{,}8806$$

falls $p_1 \uparrow \Rightarrow V_2 \uparrow \Rightarrow V_{ein} \downarrow \Rightarrow \eta_v \downarrow$

falls $V_{sch} \downarrow \Rightarrow V_2 \downarrow \Rightarrow V_{ein} \uparrow \Rightarrow \eta_v \uparrow$

(67) $Q_{zu} = 100\ kJ$

$Q_{ab} = 64\ kJ$



$$\underbrace{U_2 - U_1}_{U_2 = U_1} = 0 = Q + L \Rightarrow L = -Q = -\left(|Q_{zu}| - |Q_{ab}|\right) = -36\ kJ$$

$$\eta_{th} = \frac{|L|}{|Q_{zu}|} = 0{,}36$$

(68) $P_K = 6000\ kW$ $\qquad \dot{Q}_{zu} = 29{,}2 \cdot 10^6\ \frac{kJ}{h} = 8111{,}11\ kW$

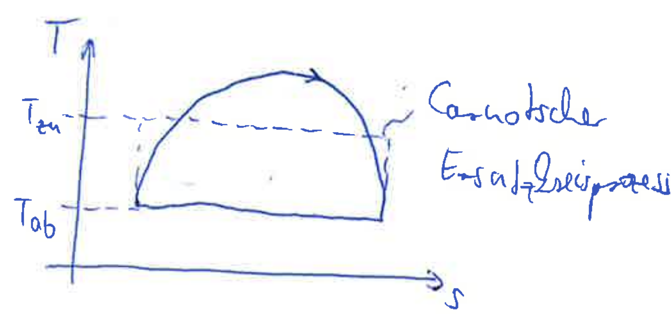
$P_T = 8000\ kW$

$$\dot{H}_2 - \dot{H}_1 = 0 = \dot{Q} + P = \dot{Q} + P_T - P_K \Rightarrow \eta_{th} = \frac{|P_T - P_K|}{|\dot{Q}_{zu}|} = 0{,}2466$$

$P = P_T - P_K = 2000\ kW$

(69) $Q_{zu} = 3550\ kJ$ $\qquad T_{ab} = 32\ °C$

$Q_{ab} = 2130\ kJ$



$$-L = |Q_{zu}| - |Q_{ab}| = 1420\ kJ$$

$$\eta_{th} = \frac{|L|}{|Q_{zu}|} = 0{,}4$$

$$\eta_{th} = 1 - \frac{T_{ab}}{T_{zu}} \Rightarrow T_{zu} = \frac{T_{ab}}{1 - \eta_{th}} = 235{,}33\ °C$$

(70) $\dot{Q}_{zu} = 800 \text{ MW}$ $\qquad T_m = 550 \text{ K}$

$\dot{Q}_{ab} = 470 \text{ MW}$ $\qquad T_u = 293 \text{ K}$

a) $-P_{el} = |\dot{Q}_{zu}| - |\dot{Q}_{ab}| = 330 \text{ MW}$

$$\eta_{th} = \frac{|P_{el}|}{|\dot{Q}_{zu}|} = 0{,}4125$$

b) $$\dot{S}_{Kessel} = \frac{\dot{Q}_{zu}}{T_m} = 1{,}45455 \ \frac{\text{MW}}{\text{K}}$$

c) $$\Delta \dot{S} = \frac{\dot{Q}_{ab}}{T_{ab}} - \frac{\dot{Q}_{zu}}{T_m} = 0{,}14955 \ \frac{\text{MJ}}{\text{s}}$$

d) $$\eta_{C} = 1 - \frac{T_{ab}}{T_{zu}} = 0{,}467273$$

(71) $\dot{Q}_{zu} = 60 \text{ kW}$, $\dot{Q}_{ab} = 100 \text{ kW}$ } in dem Standpunkt der Maschine

$T_{zu} = -33\,^\circ\text{C}$

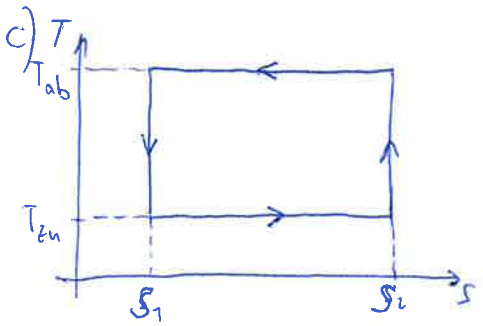
$T_{ab} = 87\,^\circ\text{C}$

a) $|\dot{L}| = |\dot{Q}_{ab}| - |\dot{Q}_{zu}| = 40 \text{ kW}$

b) $$\varepsilon_{WP} = \frac{|\dot{Q}_{ab}|}{|\dot{L}|} = 2{,}5$$

$$\varepsilon_{KM} = \frac{|\dot{Q}_{zu}|}{|\dot{L}|} = 1{,}5$$

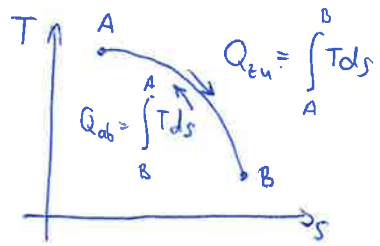
c)



$$\varepsilon_{WP} = \frac{|\dot{Q}_{ab}|}{|\dot{L}|} = \frac{T_{ab}(S_2 - S_1)}{(T_{ab} - T_{zu})(S_2 - S_1)} = \frac{T_{ab}}{T_{ab} - T_{zu}} = 3$$

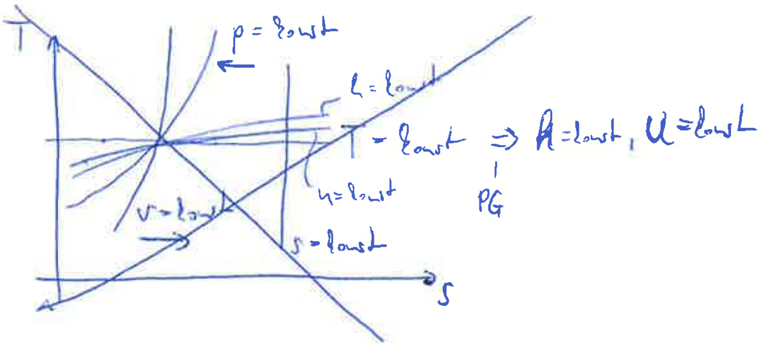
$$\varepsilon_{KM} = \frac{|\dot{Q}_{zu}|}{|\dot{L}|} = \frac{T_{zu}}{T_{ab} - T_{zu}} = 2$$

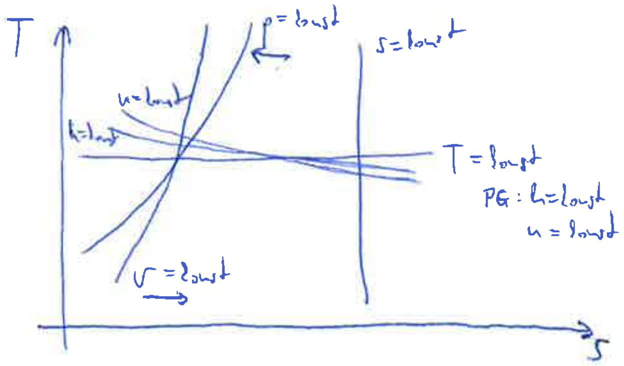
(72)



$Q_{zu} = -Q_{ab} \Rightarrow |L| = 0$

(73)





(74) $c_p = 1 \frac{kJ}{kg\,K}$ $\quad R = 287 \frac{J}{kg\,K}$ $\quad T \in [0°C, 400°C]$

$$s(T = 0°C, p = 1\,bar) = 0$$

$$s(T,p) = c_p \ln \frac{T}{T_0} - R \ln \frac{p}{p_0} \quad , \quad T_0 = 273\ K, \quad p_0 = 1\ bar$$

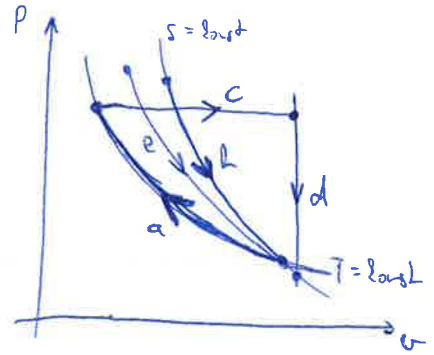
$$c_p \ln \frac{T}{T_0} = s + R \ln \frac{p}{p_0}$$

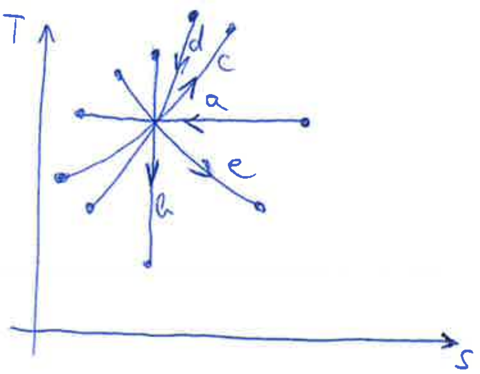
$$T(s,p) = T_0\, e^{\frac{1}{c_p}\left(s + R \ln \frac{p}{p_0}\right)} = T_0\, e^{\frac{s}{c_p}}\, e^{\frac{R}{c_p} \ln \frac{p}{p_0}}$$

$$s(T,v) = c_v \ln \frac{T}{T_0} + R \ln \frac{v}{v_0} \qquad T_0 = 273\ K$$

$$T(s,v) = T_0\, e^{\frac{1}{c_v}\left(s - R \ln \frac{v}{v_0}\right)} \qquad v_0 = \frac{T_0 R}{p_0} = 0{,}78351 \frac{m^3}{kg}$$

(75)





(76) $pv = RT$

$$u = u(v,T) \qquad du = \frac{\partial u}{\partial v}\Big|_T dv + \frac{\partial u}{\partial T}\Big|_v dT = \frac{\partial u}{\partial v}\Big|_T dv + c_v\, dT$$

$$\frac{\partial u}{\partial v}\Big|_T = -p + T \frac{\partial p}{\partial T}\Big|_v = -\frac{RT}{v} + T\frac{R}{v} = 0$$

$$du = 0 \cdot dv + c_v\, dT \Rightarrow du = c_v\, dT$$

(77) $pv = RT$

$h = h(p,T) \qquad dh = \left.\frac{\partial h}{\partial p}\right|_T dp + \left.\frac{\partial h}{\partial T}\right|_p dT = \left.\frac{\partial h}{\partial p}\right|_T dp + c_p dT$

$$\left.\frac{\partial h}{\partial p}\right|_T = v - T\left.\frac{\partial v}{\partial T}\right|_p = \frac{RT}{p} - T\frac{R}{p} = 0$$

$$dh = 0 \cdot dp + c_p dT \Rightarrow dh = c_p dT$$

(78) $u = u(T) \Rightarrow p f(v) = RT$

$u = u(v,T) \Rightarrow du = \left.\frac{\partial u}{\partial v}\right|_T dv + c_v dT$

$$\left.\frac{\partial u}{\partial v}\right|_T = -p + T\left.\frac{\partial p}{\partial T}\right|_v \underset{u=u(T)}{=} 0 \Rightarrow \left.\frac{\partial p}{\partial T}\right|_v = \frac{p}{T} \rightarrow \text{isochore Linien}$$

$$\int \left.\frac{dp}{p}\right|_v = \int \left.\frac{dT}{T}\right|_v$$

$$\ln p = \ln T + \ln K(v)$$

$$p = K(v)\, T$$

$$K(v) := \frac{R}{f(v)} \Rightarrow p f(v) = RT$$

(79) $h = h(T) \Rightarrow f(p)\, v = RT$

$h = h(p,T) \Rightarrow dh = \left.\frac{\partial h}{\partial p}\right|_T dp + c_p dT$

$$\left.\frac{\partial h}{\partial p}\right|_T = v - T\left.\frac{\partial v}{\partial T}\right|_p \underset{h=h(T)}{=} 0 \Rightarrow \left.\frac{\partial v}{\partial T}\right|_p = \frac{v}{T} \rightarrow \text{isobare Linien}$$

$$\int \left.\frac{dv}{v}\right|_p = \int \left.\frac{dT}{T}\right|_p$$

$$\ln v = \ln T + \ln K(p)$$

$$v = K(p)\, T$$

$$K(p) := \frac{R}{f(p)} \Rightarrow f(p) \cdot v = RT$$

(80) $\left.\begin{array}{l} u = u(T) \\ h = h(T) \end{array}\right\} \Rightarrow \quad pv = RT$

$$\left.\begin{array}{l} u = u(T) \Rightarrow \quad p f(v) = RT \\ h = h(T) \Rightarrow \quad g(p)\cdot v = RT \end{array}\right\} \Rightarrow p\cdot f(v) = g(p)\cdot v \Rightarrow pv = RT$$

(81) $v = \dfrac{RT}{p} - a\left(\dfrac{d}{T}\right)^3 - (b + cp^2)\left(\dfrac{d}{T}\right)^{11}$

$M = 17\ \frac{kg}{kmol}$

$a = 0{,}42\ \frac{m^3}{kg}$ $\quad b = 60\ \frac{m^3}{kg}$ $\quad m = 1\,kg$ $\quad T = 30\,^\circ C$

$c = 1{,}497\cdot 10^{-11}\ \frac{m^7}{kg N^2}$ $\quad d = 100\,K$ $\quad p_1 = 2\,bar$

$p_2 = 8\,bar$

$$h = h(p, T) \Rightarrow dh = \frac{\partial h}{\partial p}\Big|_T dp + \underbrace{c_p\, dT}_{=0\ /isotherm/} = \left(v - T\frac{\partial v}{\partial T}\Big|_p\right) dp$$

$$h_2 - h_1 = \int_{p_1}^{p_2}\left(v - T\frac{\partial v}{\partial T}\Big|_p\right) dp = -38573{,}9\ \frac{J}{kg}$$

$$u_2 - u_1 = (h_2 - p_2 v_2) - (h_1 - p_1 v_1) = (h_2 - h_1) - (p_2 v_2 - p_1 v_1) =$$
$$= -29294{,}7\ \frac{J}{kg}$$

$$s = s(p, T) \Rightarrow ds = \frac{\partial s}{\partial p}\Big|_T dp + \underbrace{\frac{c_p}{T} dT}_{=0\ /isotherm/} = -\frac{\partial v}{\partial T}\Big|_p dp$$

$$s_2 - s_1 = -\int_{p_1}^{p_2}\frac{\partial v}{\partial T}\Big|_p dp = -775{,}005\ \frac{J}{kg K}$$

$$q_{12} = \int_{s_1}^{s_2} T\, ds \underset{isotherm}{=} T(s_2 - s_1) = -234{,}827\ \frac{kJ}{kg}$$

$$l_{12} = (u_2 - u_1) - q_{12} = 205{,}532\ \frac{kJ}{kg}$$

// $l_{12,t} = \int_{p_1}^{p_2} v\, dp \Rightarrow q_{12} = (h_2 - h_1) - l_{12,t}$, $\ l_{12} = l_{12,t} - (p_2 v_2 - p_1 v_1)$ //

(82) $T = 300\ K$ $\quad v_1 = 0{,}5\ \frac{m^3}{kg} = \frac{1}{\rho_1}$ $\quad \rho_2 = 10\rho_1 \Rightarrow v_2 = \frac{v_1}{10}$

$pv = RT + Bp$ $\quad R = 189\ \frac{J}{kg\,K}$ $\quad B = (-9{,}7 + 0{,}023\,T)\,10^{-3}\ \frac{m^3}{kg}$

a) $p(v - B) = RT \Rightarrow p_2 = \left(\frac{v_2 - B_2}{RT_2}\right)^{-1} = 10{,}7386\ bar$

b) $u = u(v,T) \Rightarrow du = \underbrace{\frac{\partial u}{\partial v}\Big|_T}_{= -p + T\frac{\partial p}{\partial T}\big|_v} dv + \underbrace{\frac{\partial u}{\partial T}\Big|_v dT}_{=0\ /\text{isotherm}/}$

$$u_2 - u_1 = \int_{v_1}^{v_2} \left(-p + T\frac{\partial p}{\partial T}\Big|_v\right) dv = -18719{,}1\ \frac{J}{kg}$$

$$l_{12} = -\int_{v_1}^{v_2} p\, dv = 8312{,}42\ \frac{J}{kg}$$

$$q_{12} = (u_2 - u_1) - l_{12} = -27031{,}5\ \frac{J}{kg}$$

c) $s = s(v,T) \Rightarrow ds = \underbrace{\frac{\partial s}{\partial v}\Big|_T}_{= \frac{\partial p}{\partial T}\big|_v} dv + \underbrace{\frac{\partial s}{\partial T}\Big|_v dT}_{=0\ /\text{isotherm}/}$

$$s_2 - s_1 = \int_{v_1}^{v_2} \frac{\partial p}{\partial T}\Big|_v dv = -90{,}1052\ \frac{J}{kg\,K} = \frac{q_{12}}{T}$$

(83) $p_1 = 0{,}08\ bar$ $\quad T_1 = 40\ °C$ $\quad \beta = \chi = 0 \Rightarrow v = 1{,}005\cdot 10^{-3}\ \frac{m^3}{kg}$

$p_2 = 120\ bar$ $\quad T_2 = 41{,}3\ °C$ $\quad c_v = 4{,}18\ \frac{kJ}{kg\,K}$

$$l_{12,t} = \int_{p_1}^{p_2} v\, dp = v(p_2 - p_1) = 12{,}052\ \frac{kJ}{kg}$$

$$s_2 - s_1 = c_v \ln\frac{T_2}{T_1} + R\ln\frac{v_2}{v_1} = c_v \ln\frac{T_2}{T_1} = 17{,}3251\ \frac{kJ}{kg\,K}$$

(84) $p_1 = 10 \text{ bar}$ $T_1 = 300\ °C$ $R = 287 \frac{J}{kg K}$

$p_2 = 1 \text{ bar}$ $T_2 = 300\ °C$

$$v_2 = \frac{RT_1}{p_2} = 1,64451 \frac{m^3}{kg}$$

$$v_1 = \frac{RT_1}{p_1} = 0,164451 \frac{m^3}{kg}$$

$$l_{12} = -\int_{v_1}^{v_2} p\,dv = -\int_{v_1}^{v_2} \frac{RT}{v} dv = -RT \ln \frac{v_2}{v_1} = -378,662 \frac{kJ}{kg} = -q_{12}$$

$$s_2 - s_1 = -R \ln \frac{p_2}{p_1} = 660,842 \frac{J}{kg K}$$

(85) $T_1 = 0\ °C$ $v = konst$ $c = 4,17 \frac{kJ}{kg K}$

$T_2 = 50\ °C$

$$s_2 - s_1 = c_v \ln \frac{T_2}{T_1} = 701,313 \frac{J}{kg K}$$

(86) $p_1 = 10 \text{ bar}$ $T_1 = 100\ °C$ $M = 44 \frac{kg}{kmol}$ $\varkappa = 1,31$

$p_2 = 2 \text{ bar}$ $T_2 = 800\ °C$

$$s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1} = 1147,87 \frac{J}{kg K}$$

$$c_p = \frac{\varkappa R}{\varkappa - 1} = \frac{\varkappa \mathcal{R}}{M(\varkappa-1)} = 798,524 \frac{J}{kg K}$$ $$R = \frac{\mathcal{R}}{M} = 188,964 \frac{J}{kg K}$$

(87) a) $m = 2 kg$ $p = konst$ $T_1 = 300 K$ $T_2 = 500 K$

$R = 287 \frac{J}{kg K}$ $c_v = 717 \frac{J}{kg K}$ $c_p = c_v + R = 1004 \frac{J}{kg K}$

$$s_2 - s_1 = c_p \ln \frac{T_2}{T_1} = 512,869 \frac{J}{kg K}$$

b) $v_1 = 0,87 \frac{m^3}{kg}$ $v_2 = 0,087 \frac{m^3}{kg}$ $T_1 = 300 K$ $T_2 = 700 K$

$$s_2 - s_1 = c_v \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1} = -53,3294 \frac{J}{kg K}$$